Control of an Industrial Static VAr Compensator

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Declaration of Authorship

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- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this dissertation is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Roberto García Rochín  
Monterrey, Nuevo León, May, 2018
Dedication

To Roberto and Tano.
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Static VAr Compensators (SVC) are power devices that can modify the reactive power in different ways, in order to compensate either reactive power or line voltages, depending on the application at hand. For instance, SVC include thyristor controlled reactors and thyristor controlled switched capacitors that can compensate reactive power produced by variable predominantly inductive/capacitive loads. Due to the increasing number of variable loads and the new regulations in the Mexican electrical grid, these devices have acquired the attention of the Power Quality firms such as Diram, S.A. de C.V. Therefore, they sponsored the industrial SVC controller development.

In this thesis, it is developed a closed-loop implementation of an industrial SVC. It compensates for the reactive power under sinusoidal and balanced conditions using the quadrature currents as the feedback signal. Also, the main power theories found in literature are reviewed and it is presented a formal definition of the reactive power using an optimization problem with constrains. Then, it is used this formal definition to obtain the main AC power theories.
Acronyms and Notation

Acronyms

AC Alternating Current
ADC Analog to Digital Converter
CFE Comisión Federal de Electricidad
D-STATCOM Distribution Static Synchronous Compensator
DC Direct Current
DSP Digital Signal Processor
FC Fixed Capacitors
FHF Fixed Harmonics Filter
FSM Finite State Machine
IEEE Institute of Electrical and Electronics Engineers
RMS Root Mean Square
RTOS Real-Time Operating System
SVC Static VAr Compensator
TCR Thyristor Controlled Reactor
THD Total Harmonic Distortion
TSC Thyristor Switched Capacitor

Notation
\( \ell_2 \) Set of square-summable infinite sequences

\( \forall \) For all

\( \langle \cdot, \cdot \rangle \) Inner product

\( |\cdot| \) Complex modulus

\( \|\cdot\| \) Norm

\( \mathbb{C} \) Set of complex numbers

\( \mathbb{R} \) Set of real numbers

\( \mathbb{R}^w \) Space of real vectors with \( w \) dimension

\( X \) Complex number

\( X(t) \) Vector valued function

\( x(t) \) Complex function

\( \overline{X} \) Complex conjugate of \( X \)

\( \triangleq \) Defined as

\( A^\top \) Transpose of the vector \( A \)

\( L_2 \) Set of square-integrable functions

\( L_2^n \) Set of square-integrable vector valued functions of dimension \( n \)
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Chapter 1

Introduction

1.1 Motivation

Due to the electrical reform that was approved in 2013 in Mexico, the regulations for the interconnection of industrial loads to the transmission system and the way the reactive power is measured over a month have changed [1]. With the past regulation, the Comisión Federal de Electricidad (CFE), was the only firm in charge of the Mexican utility [2]. There was no major regulation for the interconnection of industrial loads to the transmission system and the reactive power was measured as an average over the whole month.

In the new regulation, the reactive, active and apparent power is measured every five minutes. For the interconnection of industrial loads to the transmission network, the load must have a power factor in the range of 0.95 inductive and 1, the 95% of the month period until 2026 when the power factor range will be changed to 0.97 inductive to 1, and it must be in that range the 97% of the month period [1]. If these industrial consumers do not achieve this requirements they will be charged because of non-fulfillment of the Mexican regulation [1].

On account of these changes in the regulation, power quality firms such as Diram, S.A. de C.V. are concerned about the economical and interconnection problems that the new regulations will generate to the industrial consumers, since fixed shunt compensators, such as the Fixed Capacitors (FC) and Fixed Harmonic Filters (FHF) are not enough to achieve the power factor in the established ranges, static shunt compensators such as the industrial Static Var Compensator (SVC) or the Distribution Static Synchronous Compensator (D-STATCOM) are hence used. But the industrial SVC would be the best option for most of the industrial consumers, because of economical reasons [3]. On behalf of that, Diram, S.A. de C.V. is developing a low voltage industrial SVC. The prototype will be used as a testbench for future developments and to demonstrate to costumers how the industrial SVC compensates the reactive power in a distribution network.

The topology used in the testbench developed by Diram is a delta connected TCR with four single tuned FHF, where the capacitors are connected in delta. Previous work in the control of the industrial SVC has shown that this device can compensate the reactive power using phasors and variable susceptances [4] and employing an optimization problem without restrictions [5]. These controllers use different definitions of the reactive power to be compensated, since there is no consensus if the reactive power must be computed in the time domain or in
CHAPTER 1. INTRODUCTION

the frequency domain or if it must be a periodical or an instantaneous quantity [6, 7, 8]. Thus, a suitable reactive power definition and control strategy need to be investigated.

1.2 Industrial Static VAr Compensator

Static shunt compensators are widely used in the heavy industry such as electric arc furnace and rolling mills to compensate for the reactive power produced by these fluctuating loads [9]. The benefit of the installation of these devices in a distribution network are [9]:

- Reduction of the facturation charges due to the reactive power demand by industrial loads.
- Reduction of the distribution losses.
- Liberation of the distribution network capacity.

One of these compensators is the industrial SVC. The industrial SVC is a combination of a Thyristor Controlled Reactor (TCR) and FC, FHF or Thyristor Switched Capacitors (TSC) [9] that is connected to a distribution network to compensate for the reactive power produced by the industrial loads. The topology of the industrial SVC using a TCR, FC, FHF and TSC being fed by the line currents $i_A$, $i_B$ and $i_C$ is depicted in Fig. 1.1.

![Figure 1.1: Topology of an SVC with a TCR, FHF, FC and TSC.](image)

Because of the thyristors used in the TCR, the industrial SVC has an economic advantage with respect to other static shunt compensators, such as the D-STATCOM [3] and a better compensation time response compared with the FC and FHF [4]. The disadvantages of the industrial SVC are the harmonics generated by the TCR and the impossibility to compensate harmonic currents generated by the industrial load if harmonics filters are not added to the compensator [3].
1.3 Problem Statement

In this thesis, reactive power is studied in a mathematical way. Moreover, control boards for an experimental setup are developed and programmed. The study of the reactive power should result in a formal definition of the reactive power when the active and apparent powers are given by integrals over a finite period. The relation of this definition with other main power theories is explained.

The controller implementation includes the development of the control circuits for an industrial SVC that will operate at line to line voltage of 220 V/60 Hz. These circuits should be modular, so they can be easily upgradable in the future and the overall control circuits need to be a low budget implementation. In the second part of the implementation step, the industrial SVC will be programmed to operate under sinusoidal and balanced conditions. The implementation will be evaluated on the testbench developed by Diram.

1.4 Thesis Organization

The outline of the thesis is as follows:

- Chapter 2 deals with the definition of the reactive power used in this thesis. The major power theories are briefly explained. Then, the definition of the reactive power used in this thesis is given. Along with a comparison with the major power theories.

- Chapter 3 gives an explanation of the industrial SVC working principle and the controllers found in the literature. Finally, the control algorithm for the industrial SVC under sinusoidal and balanced conditions is given.

- Chapter 4 describes the control implementation and it is divided into two parts. The first part deals with the hardware development and the second part with the programming of the Digital Signal Processor (DSP) using a Real-Time Operating System (RTOS).

- Chapter 5 gives the results of the implementation Providing open loop and closed loop data of the experiments.

- Chapter 6 gives the conclusions of the thesis and the future work.
Chapter 2

Reactive Power Definition

In this chapter, we introduce the definition of reactive power that will be used in the following chapters. In section 2.1 we give a brief discussion of the main power theories. Then, we give the reactive power definition for this thesis in section 2.2. In section 2.3 the main power theories are derived. Finally, the results are discussed in section 2.4.

2.1 Power theories

Since the industrial Static VAr Compensator (SVC) is an electrical device used to compensate the power factor in some industrial loads [4], appropriate power theories for AC systems need to be investigated in order to describe its compensation mechanism.

These power theories are calculated in the time domain or in the frequency domain and in a instantaneous or periodical manner. We give a brief description of the classical power theory with an introduction to the history of the power theory problem in AC systems [10], and also an introduction to the following power theories:

- Budeanu’s power theory [11]
- Fryze’s power theory [12]
- Current physical component [13, 14]
- Instantaneous power theories [7, 15]
- Power theory based on an optimization problem [16]
- Power theory based on vector spaces [17]

The reason why there are different methods to calculate the reactive power is because there is no consensus among electrical engineers of what the reactive power is when the voltages and currents are not sinusoidal. This has lead to vivid discussions and even changes in the IEEE 1459-2010 standard.

At the end of the section, we will give a summary of the benefits of each of the theories over the other ones.
2.1.1 Classical power theory

This was the first definition of power in AC electrical systems [18]. Its idea is to treat the power as an average value over a period $T$ that depends on the frequency of the electrical grid,

$$P \triangleq \frac{1}{T} \int_{0}^{T} v(t)i(t)dt.$$  (2.1)

Here the averaged power $P$ was named active power. To simplify the expression (2.1), it is assumed that the system is linear, driven by a sinusoidal voltage and that it is in steady state [18]. Since the differential equations that describe the circuit are linear with constant coefficients and the input (2.2) is a sinusoidal,

$$v(t) = v_{\text{max}} \cos(\omega t + \phi_v) = \sqrt{2} V_c \cos(\omega t + \phi_v),$$  (2.2)

with

$$V_c = \sqrt{\frac{1}{T} \int_{0}^{T} v(t)^2 dt}.$$  

Its output (2.3) is a phase shifted weighted sinusoidal,

$$i(t) = i_{\text{max}} \cos(\omega t + \phi_i) = \sqrt{2} I_c \cos(\omega t + \phi_i),$$  (2.3)

with

$$I_c = \sqrt{\frac{1}{T} \int_{0}^{T} i(t)^2 dt}.$$  

Therefore, the product (2.4) is the instantaneous power of the system,

$$v(t)i(t) = V_c I_c \cos(\phi_v - \phi_i)(1 + \cos(2\omega t + 2\phi_i)) + V_c I_c \sin(\phi_v - \phi_i) \sin(2\omega t + 2\phi_i).$$  (2.4)

Here, $V_c$ is the Root Mean Square (RMS) value of the voltage applied to the circuit, $\phi_v$ is the phase angle of the voltage, $I_c$ is the RMS value of the current, $\omega$ is the angular frequency of the driving function and $\phi_i$ is the phase angle of the current.

Since the product (2.4) has a constant and a time varying term, the integral is taken over the period $T$ of the alternating voltage to obtain the active power,

$$P_c \triangleq V_c I_c \cos(\phi_v - \phi_i).$$  (2.5)

Where $P_c$ will be named in this work as the active power in the classical sense, it could be positive if the load is consuming power or negative if the load is behaving as a source [18].

The amplitude of the varying term in (2.4) is defined as another power that has no contribution to the active power [18],

$$Q_c \triangleq V_c I_c \sin(\phi_v - \phi_i).$$  (2.6)
In this thesis $Q_c$ will be defined as the *reactive power in the classical sense*, its average value is zero and it is interpreted as an oscillation of energy between the load and the source. In linear circuits, this power is non zero in the presence of capacitors and inductors. Therefore, in linear circuits with purely resistive loads, the reactive power $Q_c$ vanishes [18].

Using the sine and cosine from the active (2.5) and reactive (2.6) powers in the classical sense. The *power triangle* is obtained and it is depicted in Fig. 2.1 [18],

![Power Triangle](image)

**Figure 2.1:** Power triangle for the classical power theory.

From Fig. 2.1 the *apparent power in the classical sense* $S_{\text{classical}}$ is defined as,

$$S^2_c = P^2_c + Q^2_c = V^2_c I^2_c.$$  

(2.7)

The apparent power in the classical sense $S_c$, is a quadrature addition of the active and reactive power. And also from the Fig. 2.1 the the power factor definition is [18],

$$\lambda_c = \frac{P_c}{S_c}.$$  

(2.8)

Equation (2.8) can be simplified with the definitions (2.5) and (2.7), giving the following relation,

$$\lambda_c \triangleq \cos(\phi_v - \phi_i).$$  

(2.9)

Equation (2.9) give us an idea of the phase difference between the current and voltage, and depending on the difference $\phi_v - \phi_i$, the system is treated as resistive, inductive, capacitive or a combination of resistive-inductive or resistive-capacitive.

There is another way to represent the quantities (2.5)-(2.8) by using a *phasorial* representation of the voltages and currents [19]. The phasorial representation uses complex exponential to represent the RMS value and phase angle of a sinusoidal function [19]. The phasor representation of the voltage $v(t)_c$ (2.2) and current $i(t)_c$ (2.3) are given by,

$$V_{cph} = V_c e^{j\phi_v},$$  

(2.10)

$$I_{cph} = I_c e^{j\phi_i}.$$  

(2.11)

Here, the meaning of $\phi_v, \phi_i, V_c$ and $I_c$ are the same as in (2.2)-(2.3).
CHAPTER 2. REACTIVE POWER DEFINITION

The main reason to use the phasorial representation is the computation of the apparent, active and reactive power in linear circuits [19]. To compute $S_c$, we need to perform the following calculations [19],

$$S_c = V_{cph}I_{cph},$$  \hspace{1cm} (2.12)

$$S_c = \sqrt{S_{\text{classical}}^2 + Q_{\text{classical}}^2},$$ \hspace{1cm} (2.13)

We can compute the $P_c$ and $Q_c$ from $S_c$ by taking the real part of $S_c$, that is $P_c$ and the imaginary part is $Q_c$,

$$S_c = P_c + jQ_c.$$ \hspace{1cm} (2.14)

From (2.14) we can obtain the power triangle depicted in Fig. 2.1 and the power factor (2.8) [19]. Thus, here we used two different representation of voltages and currents and we obtained the same results.

History of the power theory problem

The problem of how the power might be defined in AC systems began with a paper written by Charles P. Steinmetz in 1892 [10], where he exposed an AC arc which has a nonlinear relation between the voltage and current. Although they were in phase, the power factor (2.8) was lower than one, leading to the misunderstanding that has not been solved yet [6]. The voltages and currents obtained by Steinmetz are depicted in Fig. 2.2, where the current is a sinusoid but the voltage is not.

![Figure 2.2: Plot of the AC arc.](image-url)
2.1. POWER THEORIES

In Fig. 2.2 Steinmetz calculated that the RMS value of the voltage is 42 V, the RMS value of the current is 8.86 A, and the active power is 314 W [10]. Then performing the calculation of the power factor (2.8), he obtained,

\[ \lambda_c = \frac{314\text{W}}{(42\text{V})(8.86\text{A})} = 0.843. \] \hspace{1cm} (2.15)

This could be interpreted as a phase shift of 32°, but in the Fig. 2.2 this is not true [10].

2.1.2 Budeanu’s power theory

This theory was published in 1927 and it was the first power theory that tried to explain the problem described by Steinmetz in 1892. It was developed for a single phase system and non sinusoidal voltages and currents [11, 18]. Budeanu obtained the following relation for the active, reactive and apparent powers,

\[ P_B = \frac{1}{T} \int_0^T v(t)B i(t)B dt \triangleq \sum_{h=1}^{\infty} |V_h| |I_h| \cos(\phi_{V_h} - \phi_{I_h}), \] \hspace{1cm} (2.16)

\[ Q_B \triangleq \sum_{h=1}^{\infty} |V_h| |I_h| \sin(\phi_{V_h} - \phi_{I_h}), \] \hspace{1cm} (2.17)

\[ S_B \triangleq \sqrt{\sum_{h=1}^{\infty} |V_h|^2 \sum_{h=1}^{\infty} |I_h|^2}. \] \hspace{1cm} (2.18)

Here, \( h \) is the harmonic number, \( |V_h| \) and \( |I_h| \) is the RMS value of each voltage and current harmonics, \( \phi_{V_h} \) is the phase angle of the voltage of the \( h \)th harmonic, \( \phi_{I_h} \) is the phase angle of the current of the \( h \)th harmonic. \( P_B, Q_B \) and \( S_B \) are the active, reactive and apparent power defined by Budeanu respectively.

Since the power triangle was not accomplished with the active, reactive and apparent power derived by Budeanu, he introduced a new term named distortion power \( D_B \). By definition, it is,

\[ D_B \triangleq \sqrt{S_B^2 - P_B^2 - Q_B^2}. \] \hspace{1cm} (2.19)

The graphical relation between these powers defined by Budeanu is depicted in Fig. 2.3.

2.1.3 Fryze’s power theory

In 1931, Fryze introduced his power theory without the use of the Fourier series and in the time domain instead of the frequency domain, for non sinusoidal voltages and currents [12]. This theory was developed for single phase systems and was the first one that used the Cauchy-Schwarz inequality. Fryze defined the power factor \( \lambda_F \) as,
CHAPTER 2. REACTIVE POWER DEFINITION

![Power prism for the Budeanu power theory.](image)

Figure 2.3: Power prism for the Budeanu power theory.

\[ \lambda_F \triangleq \frac{\frac{1}{T} \int_0^T v(t)i(t)dt}{\sqrt{\frac{1}{T} \int_0^T v(t)^2 dt} \sqrt{\frac{1}{T} \int_0^T i(t)^2 dt}}. \]  \hfill (2.20)

Here the numerator is the active power \( P \) and the denominator are the RMS values of the voltage and current.

From the the power factor definition (2.20), Fryze obtained the active current of the system \( i(t)_a \), that is proportional to the voltage \( v(t) \),

\[ i(t)_a = \frac{P}{\frac{1}{T} \int_0^T v(t)^2 dt} v(t) \]  \hfill (2.21)

He used this current to show that this term generates the same amount of active power than the whole current \( i(t) \), therefore,

\[ P = \frac{1}{T} \int_0^T v(t)i(t)dt = \frac{1}{T} \int_0^T v(t)i(t)_a dt. \]  \hfill (2.22)

From the relation in (2.22), Fryze decomposed the currents in the system as an active current \( i(t)_a \) and a reactive current \( i(t)_r \), and these two currents have the following relations,

\[ i(t) = i(t)_a + i(t)_r, \]  \hfill (2.23)

\[ \frac{1}{T} \int_0^T i(t)_a i(t)_r dt = 0. \]  \hfill (2.24)

From the decomposition in (2.23), he took the RMS value at both sides of the equation and obtained

\[ I^2 = I^2_a + I^2_r. \]  \hfill (2.25)

Here, \( I^2, I^2_a \) and \( I^2_r \) are the RMS values of the total, active and reactive currents defined by Fryze.

The definition of the power triangle by Fryze is a multiplication of the squared RMS value of the voltage at both sides of the relation (2.25),

\[ S^2_F = P^2_F + Q^2_F. \]  \hfill (2.26)
2.1. Current physical component power theory

In 1983, Czarnecki developed a theory following the idea of current decomposition developed by Fryze, but in the frequency domain [13, 14, 18]. His idea is based on the decomposition of the reactive current given by Fryze into a reactance and a scatter current,

\[ i(t) = i(t)_a + i(t)_R + i(t)_s. \]  

(2.27)

Here, the active current is given by \( i(t)_a \), the reactance current is \( i(t)_R \) and the scatter current is \( i(t)_s \).

Czarnecki followed a similar approach to Fryze, and he obtained a power prism taking into consideration the active \( P_{CPC} \), reactive \( Q_{CPC} \), scattered \( D_{CPC} \) and apparent \( S_{CPC} \) powers,

\[ S_{CPC}^2 = P_{CPC}^2 + Q_{CPC}^2 + D_{CPC}^2. \]  

(2.28)

Here \( S_{CPC}, P_{CPC}, Q_{CPC} \) and \( D_{CPC} \) are the multiplication of the RMS value of (2.27) by the RMS value of the voltage supplied to the system.

2.1.5 Instantaneous power theories

The instantaneous power theories were developed to deal with power quality issues provoked by rapidly changing loads [7]. They are based on the Clarke and Park transformations of the voltages and currents in the \( abc \) domain to the \( \alpha - \beta \) or the \( d - q \) domain. The most important instantaneous power theories are the \( p - q \) theory and the synchronous reference frame theory and they will be described below.

The \( p - q \) theory

This was the first instantaneous power theory and it was published in 1984 by Hirofumi Akagi et al. [7]. It is based in the Clarke transformation that is a change of the basis of the current and voltages from the \( abc \) basis to the \( \alpha - \beta \) basis, as it is computed below,

\[
\begin{pmatrix}
  v_\alpha \\
  v_\beta
\end{pmatrix} = \sqrt{\frac{2}{3}} \begin{pmatrix}
  1 & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\
  0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}
\end{pmatrix} \begin{pmatrix}
  v_a \\
  v_b \\
  v_c
\end{pmatrix},
\]  

(2.29)

\[
\begin{pmatrix}
  i_\alpha \\
  i_\beta
\end{pmatrix} = \sqrt{\frac{2}{3}} \begin{pmatrix}
  1 & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\
  0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}
\end{pmatrix} \begin{pmatrix}
  i_a \\
  i_b \\
  i_c
\end{pmatrix}.
\]  

(2.30)

Here the instantaneous currents and voltages in the \( \alpha \) and \( \beta \) domains are respectively \( i_\alpha, i_\beta, v_\alpha \) and \( v_\beta \), the instantaneous values of the voltages and currents are \( v_a, v_b, v_c, i_a, i_b \) and \( i_c \).
With the voltages and currents defined in the $\alpha\beta$ domain, the instantaneous active power $p_{dq}$ is obtained with a dot product and the instantaneous reactive power $q_{dq}$ with a cross product of the voltages and currents in the $\alpha - \beta$ basis,

\begin{align*}
p_{p-q} &= v_\alpha i_\alpha + v_\beta i_\beta, \quad (2.31) \\
q_{p-q} &= v_\alpha i_\beta - v_\beta i_\alpha. \quad (2.32)
\end{align*}

**Synchronous reference frame theory**

This is a modification of the $p - q$ theory, it is based on a rotating basis that is following the voltage in the $d - q$ basis, that is why it is called synchronous [18]. Because of the synchronization, the voltages and currents under sinusoidal and balanced conditions in the $abc$ basis become constants in the $d - q$ basis [18]. The voltages and currents in the $d - q$ domain are computed below using the Clarke and Park transformations,

\begin{align*}
\begin{pmatrix} v_d \\ v_q \end{pmatrix} &= \sqrt{\frac{2}{3}} \begin{pmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{pmatrix} \begin{pmatrix} 1 & -1/2 & -1/2 \\ \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix} \begin{pmatrix} v_a \\ v_b \\ v_c \end{pmatrix}, \quad (2.33) \\
\begin{pmatrix} i_d \\ i_q \end{pmatrix} &= \sqrt{\frac{2}{3}} \begin{pmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{pmatrix} \begin{pmatrix} 1 & -1/2 & -1/2 \\ \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix} \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix}. \quad (2.34)
\end{align*}

Here $v_d, v_q, i_d, i_q$ are the voltages and currents in the $d - q$ basis. The voltages and currents in the $abc$ domain are $v_a, v_b, v_c, i_a, i_b, i_c$. Finally, $\omega$ is the angular frequency of the electrical network and $t$ is the time.

Now with the transformed voltages and currents the dot product and the cross product are taken.

\begin{align*}
p_{SRF} &= v_d i_d + v_q i_q, \quad (2.35) \\
q_{SRF} &= v_d i_q - v_q i_d. \quad (2.36)
\end{align*}

Here $p_{SRF}$ is the instantaneous active power and $q_{SRF}$ is the instantaneous reactive power in the synchronous reference frame theory.

**2.1.6 Power theory based on an optimization problem**

This power theory was introduced in 1985 by the Institute of Electrical Circuit Theory and Engineering [16, 18]. It is based on an optimization problem with constrains and its solution is the active current obtained by Fryze.

The optimization problem is a minimization of a square integrable current function, keeping the active power constant,
2.1. Power Theories

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{T} \int_{0}^{T} i(t)^2 dt \\
\text{subject to} & \quad P = \frac{1}{T} \int_{0}^{T} v(t)i(t) dt \in \mathbb{R}
\end{align*}
\]  

(2.37)

To solve this minimization problem the Lagrange multipliers method is used, where the optimum current coincides with the active current defined by Fryze (2.21)[18].

2.1.7 Power theory based on vector spaces

On 1997 Niels LaWhite and his advisor Marija D. Ilić published a power theory based on the vector space decomposition of a periodic signal [17]. First they defined their problem to lie on the real \( L_2 \) function space. This set has an operation defined as an inner product between two function belonging to this set,

\[
\langle x(t), y(t) \rangle = \frac{1}{T} \int_{0}^{T} x(t)y(t) dt.
\]  

(2.38)

And the induced norm of the function is given by,

\[
\| x(t) \| = \sqrt{\langle x(t), x(t) \rangle} = \sqrt{\frac{1}{T} \int_{0}^{T} x(t)^2 dt}.
\]  

(2.39)

Because of the operations defined for this function space (2.38)-(2.39), it can express the active and apparent powers using the voltage and current going into the port [17]

\[
P_{LW} = \langle i(t), v(t) \rangle,
\]  

(2.40)

\[
S_{LW} = \| v(t) \| \| i(t) \|.
\]  

(2.41)

This function space was expressed as a linear combination of finite periodic signals [17], and thus,

\[
x(t) = \sum_{j=1}^{h} x(t)_j \phi(t)_j.
\]  

(2.42)

Then, they wrote (2.42) as a dot product between two vector valued functions,

\[
x(t) = X_{LW}(t)^\top \Phi_{LW}(t).
\]  

(2.43)

With the definitions (2.38)-(2.43), they developed a power theory based on the results obtained by Fryze in (2.26),

\[
Q_{LW} = \pm \sqrt{S^2_{LW} - P^2_{LW}}.
\]  

(2.44)

Then they obtained a reactive power in terms of the harmonic components of the voltages and currents [17],
\[ Q_{LW} = \sqrt{\sum_{j=1}^{h} \sum_{k=j+1}^{n} (V_j I_k - V_k I_j)^2}. \] (2.45)

In order to do compensation with this power theory, LaWhite et al. used a constrained optimization problem, where he constrained the value of the active power \( P_{LW} \) and optimized the reactive power value \( Q_{LW} \) in order to obtain a better power factor \( \lambda_{LW} \). Here the power factor was defined as [17],

\[ \lambda_{LW} = \frac{|P_{LW}|}{\sqrt{P_{LW}^2 + Q_{LW}^2}}. \] (2.46)

### 2.1.8 Power theory summary

In section 2.1 six power theories were briefly described. They were developed in the frequency or in the time domain and they were instantaneous or periodic, but they all try to describe the phenomena described by Steinmetz [10]. From those six, the first one to be developed was Budeanu’s power theory. Although it is the most accepted among engineers, it has also generated a lot of controversy, since it has an erroneous interpretation of energy phenomena in non-sinusoidal periodical circuits [18, 20]. Thus, in 2010 the IEEE (Institute of Electrical and Electronics Engineers) derogated it from the standard IEEE 1459-2010, but it was added in this literature review because of its historical importance.

The next theory that was developed was the one by Fryze, and it was the first using notions of a Hilbert space [21], since it used the Cauchy-Schwarz inequality to describe the phenomena. And because this theory was developed in the time domain, it is mostly used as a reference value for the controller of the active filters [18]. Since this reference value has a norm in the denominator of the active current (2.21), this can not be known until the end of the period.

The third explained theory was the CPC theory, the benefit of this theory is that it can be used to design reactive compensators under non-sinusoidal voltage conditions, since it is expressed in the frequency domain it is not really useful to control static shunt compensators [18].

The fourth theory and the most used to control D-STATCOMs is the instantaneous power theory [18], the benefit of this theory is that it gives us a value for the active and reactive power instantaneously, but this values are just valid under sinusoidal and balanced conditions in a three-phase three wire system [6].

The power theory based on an optimization problem, is not really used in the industry, however from the research point of view it give us an insight on the physical meaning of the power theory, since the solution of this isoperimetric problem is the active current obtained by Fryze [18] that was the basis for other main power theories. Although the isoperimetric problem is not related to an energy problem, rather to a constrained least squares problem [22, 23].

Finally the power theory published by LaWhite et. al., is the one describing most of the phenomena, since it can be used to describe power related of an \( n \)-phase system and it is related with the Fryze and CPC theory, since it was developed using notions of a Hilbert space.
2.2 Derivation of the reactive power

In order to obtain the derivation of a generalized reactive power, we must follow a physical interpretation of the problem. To do that, we extend the optimization problem described in Subsection 2.1.6 to a case of \( n \) phases. So the problem will be to minimize the current and keeping the energy transfer between the electric network and the industrial load constant. To express the voltages and currents, we will use real vector valued function,

\[
\mathbf{V}(t) = \begin{bmatrix} v_1(t) & v_2(t) & v_3(t) & \ldots & v_n(t) \end{bmatrix}^T, \tag{2.47}
\]

\[
\mathbf{I}(t) = \begin{bmatrix} i_1(t) & i_2(t) & i_3(t) & \ldots & i_n(t) \end{bmatrix}^T. \tag{2.48}
\]

Where \( \mathbf{V}(t) \) and \( \mathbf{I}(t) \) are the vector valued functions of the voltage with respect to ground and the line currents respectively. A graphical representation of how the voltages and currents must be measured is depicted in Fig. 2.4.

![Figure 2.4: Interconnection between an industrial load and the electrical network.](image)

With the voltages (2.47) and currents (2.48) defined, the optimization problem (2.37) is extended to the \( n \) phase case,
\[
\text{minimize} \quad \frac{1}{T} \int_{0}^{T} I(t)^\top I(t) dt,
\]
\[\text{subject to} \quad P = \frac{1}{T} \int_{0}^{T} I(t)^\top V(t) dt \in \mathbb{R}. \tag{2.49}\]

Here \(T\) is a time period and \(P\) is the active power supplied at that time period. To solve the optimization problem (2.49), we recall that (2.47)-(2.48) are real vector valued functions. In the optimization problem (2.49) the minimizing functional corresponds to the squared norm of the current vector (2.48) and the constrain to the inner product of the \(L^2_n[0, T]\), when \(a = 0\) and \(b = T\). Thus, the optimization problem (2.49) lies in the real \(L^2_n[0, T]\) space. Furthermore, the problem (2.49) has the form of a least-effort problem that is described in Appendix A.

Since the problem (2.49) is a least-effort problem, we can use Theorem 3 given in Appendix A to solve it. First, we need to find the base for \(M\). By the definition of the constrain in (2.49) we got,

\[\langle I(t), V(t) \rangle = P. \tag{2.50}\]

Thus from (2.50) the basis for \(M\) is \(\{V(t)\}\). And from Theorem 3, the minimum current vector \(I(t)_0\) must lie in \(M\). Thus,

\[I(t)_0 = \beta V(t). \tag{2.51}\]

Now, we construct the Gram equation by substituting (2.51) into (2.50),

\[\langle \beta V(t), V(t) \rangle = \frac{1}{T} \int_{0}^{T} \beta V(t)^\top V(t) dt = P. \tag{2.52}\]

From (2.52) we obtain the value of \(\beta\),

\[\beta = \frac{P}{\frac{1}{T} \int_{0}^{T} V(t)^\top V(t) dt}. \tag{2.53}\]

Thus, the unique minimum norm current from (2.49) is,

\[I(t)_0 = \frac{P}{\frac{1}{T} \int_{0}^{T} V(t)^\top V(t) dt} V(t) = \frac{P}{\|V(t)\|^2} V(t). \tag{2.54}\]

### 2.2.1 Generalized power triangle

With the least-effort problem (2.49), solved in Section 2.2. We obtained the minimum current that achieves the same energy transfer between the electric network and the industrial load. The geometrical interpretation of the solution (2.54) is depicted in Fig. 2.5.

From the Fig. 2.5 we can derive a decomposition of the current vector \(I(t)\) into two orthogonal currents, \(I(t)_0\) and \(I(t) - I(t)_0\). The current \(I(t)_0\) is the minimum current that will generate the same energy transfer between the electric network and the industrial load. For the second current \(I(t) - I(t)_0\) the inner product with the voltage will give zero, meaning that it does not contribute to the energy exchange between the load and the electrical network in the
2.2. DERIVATION OF THE REACTIVE POWER

Figure 2.5: Geometrical interpretation of the power triangle in \( \mathbb{R}^2 \).

time period \([0, T]\). Also, from Fig. 2.5 we can derive the following relation for the minimum and orthogonal current:

\[
I(t)_0 = I(t) \cos(\theta), \tag{2.55}
\]

\[
I(t) - I(t)_0 = I(t) \sin(\theta). \tag{2.56}
\]

Therefore, these two currents (2.55)-(2.56) are projections of \( I(t) \) onto the \( M \) subspace and the \( V \) hyperplane respectively. In order to obtain the norm of the total current \( I(t) \), we add (2.55)-(2.56) in quadrature,

\[
\| I(t) \|^2 = \| I(t)_0 \|^2 + \| I(t) - I(t)_0 \|^2. \tag{2.57}
\]

Equation (2.57) tells us that the orthogonal current \( I(t) - I(t)_0 \) affect the ampacity of the electrical network. Thus, this quantity must be as low as possible.

One more piece of information that we can obtain from Fig. 2.5 and the equation (2.57) is that in order to minimize \( I(t) - I(t)_0 \), we need to know its direction and its norm, so we can feed a current in the opposite direction and with the same norm to make it zero.

Another way to write (2.57) is by multiplying the left and right hand side by the squared norm of the voltage,

\[
\| V(t) \|^2 \| I(t) \|^2 = \| V(t) \|^2 \| I(t)_0 \|^2 + \| V(t) \|^2 \| I(t) - I(t)_0 \|^2. \tag{2.58}
\]

The equation (2.58) is called the power triangle. The terms in (2.58) are called the apparent \( S \), active \( P \) and reactive \( Q \) powers,
CHAPTER 2. REACTIVE POWER DEFINITION

\[ S = \| \mathbf{V}(t) \| \| \mathbf{I}(t) \|, \quad (2.59) \]

\[ P = \| \mathbf{V}(t) \| \| \mathbf{I}(t) \|_0, \quad (2.60) \]

\[ Q = \| \mathbf{V}(t) \| \| \mathbf{I}(t) - \mathbf{I}(t)_0 \|. \quad (2.61) \]

If we do not add \( Q \) (2.61) into (2.58) and we use the square root at both sides, we obtain the inequality (2.62),

\[ \| \mathbf{V}(t) \| \| \mathbf{I}(t) \|_0 \leq \| \mathbf{V}(t) \| \| \mathbf{I}(t) \|. \quad (2.62) \]

From (2.55) we rewrite (2.62) as,

\[ \| \mathbf{V}(t) \| \| \mathbf{I}(t) \| \cos(\theta) \leq \| \mathbf{V}(t) \| \| \mathbf{I}(t) \|. \quad (2.63) \]

In (2.63) the left-hand side of the inequality is the definition of the inner product (A.4). Then, the inequality (2.63) is the Cauchy-Schwarz inequality (A.3). Thus, the power triangle (2.58) is just a property of the functions used to express the voltages and currents and it gives the solution of the least-effort problem if the active power (2.60) is assumed to be constant.

Now we will derive the power triangle for the Hilbert space used here.

The real vector valued functions (2.47) and (2.48) in the problem (2.49) were constrained to be in the real \( L^2[0,T] \) space. Since this space is a Hilbert space, the Cauchy-Schwarz inequality (A.11) holds. Thus, the equation (A.13) also holds, for the real \( L^2[0,T] \) space this equation becomes,

\[
\left( \frac{1}{T} \int_0^T \mathbf{I}(t)^\top \mathbf{V}(t) dt \right)^2 + \frac{1}{2T^2} \int_0^T \int_0^T \sum_{r=1}^n \sum_{s=1}^n (v_r(t)i_s(\tau) - v_s(\tau)i_r(t))^2 dt d\tau
\]

\[ = \left( \frac{1}{T} \int_0^T \mathbf{V}(t)^\top \mathbf{V}(t) dt \right) \left( \frac{1}{T} \int_0^T \mathbf{I}(t)^\top \mathbf{I}(t) dt \right) \quad (2.64) \]

Thus, from (2.64) we obtain the definitions of the apparent, active and reactive powers,

\[ S = \sqrt{\frac{1}{T} \int_0^T \mathbf{V}(t)^\top \mathbf{V}(t) dt \sqrt{\frac{1}{T} \int_0^T \mathbf{I}(t)^\top \mathbf{I}(t) dt}}, \quad (2.65) \]

\[ P = \frac{1}{T} \int_0^T \mathbf{I}(t)^\top \mathbf{V}(t) dt, \quad (2.66) \]

\[ Q = \sqrt{\frac{1}{2T^2} \int_0^T \int_0^T \sum_{r=1}^n \sum_{s=1}^n (v_r(t)i_s(\tau) - v_s(\tau)i_r(t))^2 dt d\tau}, \quad (2.67) \]

\[ P^2 + Q^2 = S^2. \quad (2.68) \]

Equation (2.68) with the defined quantities (2.65)-(2.67) is the generalization of the power triangle given by the Classical Power Theory in (2.7). This generalization is for a \( n \) phase system with a generalized form of voltages and currents.
2.3 Comparison with the main power theories

In this section, it will be addressed how the major power theories are related to the least-effort problem (2.49).

2.3.1 Classical power theory

The classical theory assumes that the electrical network has just the fundamental component. Thus, if we would like to express the voltages and currents with complex functions, these functions would just need one element of the basis (A.28) [18]. From the relation between the complex $L_2[a, b]$ and $\ell_2$ spaces stated in Appendix A.7, we would need just one element of the complex sequence to describe the voltage and another one to represent the current. Therefore, with this defined, the power triangle is just the Fibonacci-Brahmagupta equation (A.35).

To prove this, we use the definition of the classical apparent power in its complex representation $S_{\text{classical}}$ (2.12),

$$S_{\text{classical}} = V_{\text{cph}} I_{\text{cph}}.$$  \hspace{1cm} (2.69)

Then, we expand it into its real and imaginary components,

$$V_{\text{cph}} = v_R + jv_I,$$
$$I_{\text{cph}} = i_R + ji_I,$$
$$V_{\text{cph}} I_{\text{cph}} = (v_Ri_R + v_Ii_I) + j(v_Ii_R - v_Ri_I).$$  \hspace{1cm} (2.70)

Finally we take the complex modulus on left and right of (2.70),

$$|v_R + jv_I|^2|i_R + ji_I|^2 = |v_Ri_R + v_Ii_I|^2 + |v_Ii_R - v_Ri_I|^2.$$  \hspace{1cm} (2.71)

With (2.71), we know that the classical power theory is solving a least-effort problem with the assumptions that the load has just the fundamental component.

2.3.2 Budeanu’s power theory

Budeanu’s power theory tries to obtain a relation of the power triangle using the apparent $S_B$, active $P_B$, reactive $Q_B$ and distortion power $D_B$. But it has been highly criticized since it does not capture the power phenomena [11, 18]. Now we will obtain the active and reactive values defined by Budeanu, using the Lagrange’s identity for the complex $\ell_2$ space (A.34).

To obtain the Budeanu’s components, we will use the squared inner product defined in the complex $\ell_2$ space and express the voltages and currents in its real and imaginary part,
\[
V_{Bk} = |V_{Bk}| \cos(\theta_{vk}) + j|V_{Bk}| \sin(\theta_{vk}),
\]
\[
I_{Bk} = |I_{Bk}| \cos(\theta_{ik}) + j|I_{Bk}| \sin(\theta_{ik}),
\]
\[
\sum_{k=1}^{\infty} V_{Bk}I_{Bk} = \left( \sum_{k=1}^{\infty} |V_{Bk}| \left( \cos(\theta_{vk} - \theta_{ik}) + j \sin(\theta_{vk} - \theta_{ik}) \right) \right)^2. \tag{2.72}
\]

Using the definitions (2.16)-(2.17) in (2.72),
\[
\left| \sum_{k=1}^{\infty} V_{Bk}I_{Bk} \right|^2 = \left| P_B + jQ_B \right|^2 = P_B^2 + Q_B^2. \tag{2.73}
\]

Finally, the definitions of the Budeanu’s apparent and distortion power are,
\[
S_B = \sum_{k=1}^{\infty} |V_{Bk}|^2 \sum_{k=1}^{\infty} |I_{Bk}|^2, \tag{2.74}
\]
\[
D_B = \sqrt{\sum_{1 \leq p < k < \infty} \left| V_{Bp}I_{Bk} - V_{Bk}I_{Bp} \right|^2}. \tag{2.75}
\]

Budeanu stated that the powers to be compensated are \(Q_B\) and \(D_B\) [18]. But if we compensate \(Q_B\), then it will modify the inner product (2.73) thus, violating the assumption under which the power triangle can solve for the least-effort problem. Other problem with this theory is that is does not give the direction in the infinite dimensional space in order to do the compensation. Thus, this theory does not capture the power phenomena, but because of its historical importance it was added in this thesis.

2.3.3 Fryze’s power theory

Fryze obtained his theory using the Cauchy-Schwarz inequality [12], knowing that this inequality becomes an equality when the voltage and current are proportional. He calculated this proportionality constant, but he did not give a physical interpretation of that problem. To obtain Fryze’s results with the least-effort problem (2.49), it is specified that the circuit have just one phase \(n = 1\). With this assumption, the optimal current for a phase phase system (2.54) is,
\[
i_1(t)_0 = \frac{P}{\|v_1(t)\|^2}v_1(t). \tag{2.76}
\]

Therefore, the Fryze’s current (2.76) is just a particular case of the solution of the least-effort problem (2.49) for a one phase circuit.

2.3.4 Current physical component power theory

The Current Physical Component power theory developed by Czarnecki decomposes the load current into three mutually orthogonal components in the complex \(\ell_2\) space. The first current
2.3. COMPARISON WITH THE MAIN POWER THEORIES

is the active current. And it solves the least-effort problem for a single phase system. The other two components are decomposed into a current that can be compensated by passive elements and it is called the reactance current [13]. And a third current that can not be compensated by passive elements, that is the scattered current [13]. To compensate this third current it is needed to use an active filter such as the D-STATCOM.

The main advantage of this theory is that we can easily design passive filters to improve the power factor [18], since it uses the notion of direction in a infinite dimensional space and the norm of the vector in that direction. But it does not give an interpretation of the power phenomena.

2.3.5 Instantaneous power theory

These power theories are based on the assumption that the voltages and currents are balanced and sinusoidal in a three phase system. With these assumptions we use the real $L^3_2[a, b]$ basis (A.14) to express these assumed vectors [25],

\[
\begin{pmatrix}
  v_1(t) \\
  v_2(t) \\
  v_3(t)
\end{pmatrix}
= \begin{pmatrix}
  \sqrt{2}V_{\text{ins}} \cos(\omega t) \\
  \sqrt{2}V_{\text{ins}} \cos(\omega t - \frac{2}{3}\pi) \\
  \sqrt{2}V_{\text{ins}} \cos(\omega t + \frac{2}{3}\pi)
\end{pmatrix},
\]

\[
\begin{pmatrix}
  i_1(t) \\
  i_2(t) \\
  i_3(t)
\end{pmatrix}
= \begin{pmatrix}
  \sqrt{2}I_{\text{ins}} \cos(\omega t - \theta_i) \\
  \sqrt{2}I_{\text{ins}} \cos(\omega t - \frac{2}{3}\pi - \theta_i) \\
  \sqrt{2}I_{\text{ins}} \cos(\omega t + \frac{2}{3}\pi - \theta_i)
\end{pmatrix}.
\]

Here $v_1(t), v_2(t), v_3(t), i_1(t), i_2(t), i_3(t)$ are the voltages and currents of each phase. $\omega$ is the angular frequency, $\theta_i$ is the phase difference between the voltages and currents and $V_{\text{ins}}$ and $I_{\text{ins}}$ are the RMS values of the voltages and currents with respect to ground.

Since the voltages and currents given in (2.77)-(2.78) are balanced. Thus,

\[
v_1(t) + v_2(t) + v_3(t) = i_1(t) + i_2(t) + i_3(t) = 0.
\]

The meaning of equation (2.79) is that these voltages and currents lie in the plane orthogonal to $(1, 1, 1) \in \mathbb{R}^3$. Using these geometrical insight, we derive the most common instantaneous power theories.

The $p - q$ theory

This power theory uses the Clarke transformation (2.29) to obtain the voltages and currents in the $\alpha, \beta$ coordinate system,

\[
\begin{pmatrix}
  v_\alpha \\
  v_\beta
\end{pmatrix}
= \begin{pmatrix}
  \sqrt{3}V_{\text{ins}} \cos(\omega t) \\
  \sqrt{3}V_{\text{ins}} \sin(\omega t)
\end{pmatrix},
\]

\[
\begin{pmatrix}
  i_\alpha \\
  i_\beta
\end{pmatrix}
= \begin{pmatrix}
  \sqrt{3}I_{\text{ins}} \cos(\omega t - \theta_i) \\
  \sqrt{3}I_{\text{ins}} \sin(\omega t - \theta_i)
\end{pmatrix}.
\]
With the voltages and currents given in (2.80)-(2.81), we compute the instantaneous active (2.31) and reactive (2.32) powers in the $\alpha - \beta$ coordinate system.

\[
\begin{align*}
    p_{p-q} &= 3V_{ins}I_{ins} \cos(\theta_i), \\
    q_{p-q} &= -3V_{ins}I_{ins} \sin(\theta_i).
\end{align*}
\]

(2.82)  (2.83)

The synchronous reference frame theory

The synchronous reference frame theory, uses the Clarke-Park transformation to convert the voltages and currents to the $d - q$ coordinate system,

\[
\begin{align*}
    \begin{pmatrix}
        v_d \\
        v_q
    \end{pmatrix} &= \begin{pmatrix}
        \sqrt{3}V_{ins} \\
        0
    \end{pmatrix}, \\
    \begin{pmatrix}
        i_d \\
        i_q
    \end{pmatrix} &= \begin{pmatrix}
        \sqrt{3}I_{ins} \cos(\theta_i) \\
        -\sqrt{3}I_{ins} \sin(\theta_i)
    \end{pmatrix}.
\end{align*}
\]

(2.84)  (2.85)

With these voltages (2.84) and currents (2.85) we obtain the instantaneous active (2.35) and reactive power (2.36),

\[
\begin{align*}
    p_{SRF} &= 3V_{ins}I_{ins} \cos(\theta_i), \\
    q_{SRF} &= -3V_{ins}I_{ins} \sin(\theta_i).
\end{align*}
\]

(2.86)  (2.87)

The results from (2.83) and (2.87) gives us the notion of how far we are from the optimum current when there is a phase shift between these two [15]. But it does not gives information of how to achieve the minimum current when the voltages and currents are unbalanced or contain harmonic components [6].

2.3.6 Power theory based on vector spaces

The theory developed by LaWhite et. al. is the one with the highest similarity with the theory developed in this thesis. They assumed that their functions lie in a real $L_2[0, T]$ space. With that assumption they obtained the reactive power definition (2.45) that looks just like the Lagrange’s identity given in (A.33). Then, they used an optimization algorithm to minimize (2.45) and with that obtain the minimum norm current [17].

The main difference between this theory with the one developed in this thesis is that they do not give a physical interpretation of the problem, so they just end up performing a minimization of (2.45) without giving a physical interpretation of it.

2.4 Conclusion

In this chapter we studied six major power theories and we derived a formal definition of the reactive power for the whole forms of voltages and currents using the least-effort problem.
2.4. CONCLUSION

We obtained the solution of the least-effort problem using a geometrical approach. With this approach we can gain more insight into the power theory problem. The first thing that we can extract from this solution is that the reactive power is a scaled distance in an infinite dimensional space (2.61) to an optimum solution (2.54). Thus, if we attain that optimum current we will liberate ampacity of the electrical network, so it could carry more current along its transmission and distributions networks.

The second piece of information that we can obtain from the least-effort problem is that since the used functions lie in a $L_2$ or $\ell_2$ space, this theory is also applicable to circuits that are not periodical and it can be also applied to Direct Current (DC) circuits. It is possible since the least-effort problem minimizes the used current by the system with an applied voltage, so the restrictions that we give is that the voltages and currents must lie in a Hilbert space.

Finally, since one of the implementation restrictions of the thesis is that the system is operating in balanced and sinusoidal conditions. Therefore, we can use the instantaneous power theories explained at subsection 2.3.5 to solve the least-effort problem. The instantaneous theory that will be used in this thesis to control the reactive power is the synchronous reference frame theory. This theory decomposes the current into two components. One that is in phase with the voltage and other current that is in quadrature with it. Thus, to solve the least-effort problem we must vanish the quadrature current.
Chapter 3

Industrial Static VAr Compensator Control

In this chapter, we develop the control strategy for an industrial SVC working under sinusoidal and balanced conditions. In section 3.1 we describe briefly the working principle of the FHF and the TCR. The controllers given in the literature are summarized in section 3.2. Finally, the formulation of this thesis controller is described in section 3.3.

3.1 Industrial Static VAr Compensator circuit

The power circuit used in the testbench was designed by Miguel Angel Rueda Muñiz and Carlos Carrillo Cortez at Diram in 2014. The testbench consist on a line reactor, a TCR and four FHFs fed by a three phase 220 V/60 Hz bus. The testbench topology is depicted in Fig. 3.1. And the photos of the power circuit are in Appendix B.

![Testbench schematic](image)

Figure 3.1: Testbench schematic.

The contactors of the testbench are denoted by $K$ and a subindex depending on the device that they are activating. The thermal breakers are the elements marked as $Th$ with
a subindex that depends on the activated device. The iron core reactors inductance and the
capacitors capacitance values are given in Table 3.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{LR}$</td>
<td>1 mH</td>
</tr>
<tr>
<td>$L_{TCR}$</td>
<td>14.81 mH</td>
</tr>
<tr>
<td>$L_{HF2}$</td>
<td>11.67 mH</td>
</tr>
<tr>
<td>$C_{HF2}$</td>
<td>50.1 µF</td>
</tr>
<tr>
<td>$L_{HF3}$</td>
<td>7.98 mH</td>
</tr>
<tr>
<td>$C_{HF3}$</td>
<td>37.6 µF</td>
</tr>
<tr>
<td>$L_{HF4}$</td>
<td>4.57 mH</td>
</tr>
<tr>
<td>$C_{HF4}$</td>
<td>37.6 µF</td>
</tr>
<tr>
<td>$L_{HF5}$</td>
<td>3.09 mH</td>
</tr>
<tr>
<td>$C_{HF5}$</td>
<td>37.6 µF</td>
</tr>
</tbody>
</table>

Table 3.1: Rating values of the experimental setup.

Because of the definition of the reactive power as an scaled distance to an optimal con-
dition given in Chapter 2 we will study the currents demanded by the four FHFs and the TCR.
In order to design the industrial SVC controller algorithm.

### 3.1.1 Fixed harmonic filters

The FHFs used in this thesis correspond to the single tuned harmonic filters [4]. This kind of
FHF is depicted in Fig. 3.2.

![Three phase FHF with the capacitors connected in delta.](image)

Figure 3.2: Three phase FHF with the capacitors connected in delta.

Here, $i_{aHF_n}$, $i_{bHF_n}$ and $i_{cHF_n}$ are the line currents of the $n$ FHF. $L_{HF_n}$ is the tuning
reactor and $C_{HF_n}$ are the capacitors of the FHF.

In the next subsections the consumed current of the FHF will be obtained.
Consumed steady state FHF current

The single tuned FHF, will be ideally consuming a current that will have the same frequency as the grid frequency and another one as its natural response without damping [26]. Since in the real life circuits there is always damping we will neglect the natural response of this circuit. To obtain this simplified current it is transformed the delta connected capacitors to its wye equivalent. The wye equivalent capacitors are given in Table 3.2.

<table>
<thead>
<tr>
<th>Capacitor Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{HFY2}$</td>
</tr>
<tr>
<td>$C_{HFY3}$</td>
</tr>
<tr>
<td>$C_{HFY4}$</td>
</tr>
<tr>
<td>$C_{HFY5}$</td>
</tr>
</tbody>
</table>

Table 3.2: Wye equivalent capacitors.

Thus, the simplified line currents assuming that the voltage is a cosine function are given by [26],

\[
i_{aFHF_n} = -\omega V_{LN} C_{HFYk} \left( \frac{\omega_0^2}{\omega_0^2 - \omega} \right) \sin(\omega t), \quad (3.1)
\]

\[
i_{bFHF_n} = -\omega V_{LN} C_{HFYk} \left( \frac{\omega_0^2}{\omega_0^2 - \omega} \right) \sin(\omega t - \frac{2\pi}{3}), \quad (3.2)
\]

\[
i_{cFHF_n} = -\omega V_{LN} C_{HFYk} \left( \frac{\omega_0^2}{\omega_0^2 - \omega} \right) \sin(\omega t + \frac{2\pi}{3}), \quad (3.3)
\]

\[
\omega_0 = \sqrt{\frac{1}{L_{HFk} C_{HFYk}}} \quad (3.4)
\]

Here, $\omega$ is the frequency of the grid, $V_{LN}$ is the maximum line to neutral voltage. $C_{HFYk}$, $L_{HFk}$ and $\omega_0k$ are the wye equivalent capacitor, the inductance of the tuning reactor and the natural frequency of the $k$th FHF respectively. The line currents (3.1)-(3.3) are leading the phase voltage. This is depicted in Fig. 3.3.

![Figure 3.3](image-url)

Figure 3.3: Comparison of the normalized line current and phase voltage.
3.1.2 Thyristor Controlled Reactor

The main part of the SVC is the TCR. This device has the advantage over the TSC that it can be fired from its voltage peak value to the next zero crossing of the voltage [4]. Because the reactors will not produce undesired current transients [4]. In this section we will study the working principle of the three phase TCR connected in delta as the one depicted in Fig. 3.4.

![Three phase TCR connected in delta.](image)

Figure 3.4: Three phase TCR connected in delta.

The TCR will be assumed to be driven by sinusoidal and balanced voltages. Since these are the constrains imposed to the operating voltages in Chapter 1. For the three phase TCR, the thyristors will be firing with respect to the line to line voltage and the supplied current is given by the line currents. The principle of operation is depicted in Fig. 3.5.

![Phase and line current waveforms in a delta connected TCR.](image)

Figure 3.5: Phase and line current waveforms in a delta connected TCR.
From the operating currents depicted in Fig. 3.5. We can see that they are lagged by 90°. So these currents can just compensate for leading currents as the ones produced by capacitive elements [4].

**Fundamental line current**

From Fig. 3.5 we proceed to obtain the fundamental line quantity of the TCR. To obtain this quantity we choose the time origin to coincide with the peak value of the line to line voltage. Thus, the firing angle will be in the range \([0, \frac{\pi}{2}]\). And the phase current is given by

\[
i_{ph} = \begin{cases} 
\frac{V_{LL}}{\omega L} (\sin(\omega t) - \sin(\alpha)), & \alpha < \omega t < \alpha + \sigma \\
0, & \alpha + \sigma < \omega t < \alpha + \pi.
\end{cases}
\]  

(3.5)

Here \(i_{ph}\) is the phase current of the three phase TCR, \(V_{LL}\) is the line to line voltage peak value, \(\alpha\) is the firing angle, \(\omega\) is the angular frequency, \(t\) is the time, \(L\) is the inductance of the reactor and \(\sigma\) is the conduction angle.

Now it is obtained the peak value of the fundamental frequency of (3.5),

\[
I_{1-ph} = \frac{2}{\pi} \int_{0}^{\alpha + \sigma} \frac{V_{LL}}{\omega L} (\sin(\omega t) - \sin(\alpha)) \sin(\omega t) d(\omega t),
\]

\[
= \frac{V_{LL}}{\pi \omega L} [\pi - 2\alpha - \sin(2\alpha)].
\]  

(3.6)

In order to obtain the peak value of the line current with respect to the conduction angle. We perform the following change of variables,

\[
\alpha = \frac{\pi - \sigma}{2}.
\]  

(3.7)

Here, \(\sigma\) is the conduction angle. Thus, we substitute (3.7) into (3.6),

\[
I_{1-ph} = \frac{V_{LL}}{\pi \omega L} [\sigma - \sin(\sigma)].
\]  

(3.8)

Finally we obtain the peak value of the fundamental line current from (3.8),

\[
I_{1-\ell} = \frac{\sqrt{3} V_{LL}}{\pi \omega L} [\sigma - \sin(\sigma)].
\]  

(3.9)

The per unit fundamental line current of the TCR is depicted in Fig. 3.6.

**Total harmonic distortion**

In the previous subsection we obtained the peak value of the fundamental line current. Because of the TCR working principle, it also generates harmonic components [4]. Thus, here we will study the Total Harmonic Distortion (THD) of the it. The THD of the line current is defined as
THD = $\sqrt{I_{\ell-RMS}^2 - I_{1-\ell,RMS}^2}$. \hspace{1cm} (3.10)

Here, $I_{\ell-RMS}$ is the RMS value of the line current and $I_{1-\ell,RMS}$ is the RMS value of (3.9). Thus, the quantity $\sqrt{I_{\ell-RMS}^2 - I_{1-\ell,RMS}^2}$ is the RMS value of the harmonic components. In Fig. 3.7 are depicted the RMS values of the fundamental and harmonics current component changing with the firing angle $\alpha$.

Now using these RMS values, the THD of the line current are computed. The THD with respect to the firing angle $\alpha$ is depicted in Fig. 3.8.

Since the THD of the line current depicted in Fig.3.8 is low in a certain firing angle range. Some authors assume that the delta connected TCR driven by sinusoidal and balanced voltages is a variable suceptance device [4]. In the next section will be derived the control algorithm of the industrial SVC.

### 3.2 Industrial SVC controllers

Up to this day the main approach to control an industrial SVC is the variable suceptance approach [4]. Another approach that is important because it uses an optimization problem without restriction is the optimal load compensation [5]. Each one of these control strategies will be briefly exposed in this section.
3.2. INDUSTRIAL SVC CONTROLLERS

Figure 3.7: Per unit RMS values of the fundamental and harmonics for the delta-connected TCR.

Figure 3.8: THD of the line current.
3.2.1 Variable susceptance approach

The variable susceptance approach was used for the first time by Gyugyi et al. from the Westinghouse Electric Corporation in 1976 [9]. Where they abstracted the three phase TCR connected in delta as a variable susceptance reactor. With this approach they developed three kind of controllers, a feedforward controller, closed-loop controller and a mixture of those two [4]. In this subsection we will just explain the closed-loop controller because is the one with the most similarities to the used in this thesis.

The closed-loop controller is used to maintain a quantity in a prescribed level. This is mostly used in industrial SVCs along with a forward controller in order to have a fast response. This controller uses phasorial analysis in order to achieve the compensation. Thus, it will be performing Fourier Analysis to obtain the magnitude and phase of the phasors. The diagram for the industrial SVC controller is depicted in Fig. 3.9.

![Diagram of Industrial SVC Controller](image)

Figure 3.9: Closed-loop control for power factor compensation.

3.2.2 Optimal load compensation

The optimal load compensation is based on an optimization problem without restrictions [5]. This compensation strategy assumes that the voltage is sinusoidal

\[ v(t) = V \cos(\omega t). \]  \hspace{1cm} (3.11)

Here, \( v(t) \) is the voltage depicted in Fig. 3.10, \( V \) is the peak voltage and \( \omega \) is the angular frequency. The functional to be compensated is [5]

\[ J_1 = \int_0^T i^2(t) dt. \]  \hspace{1cm} (3.12)

In (3.12), \( i(t) \) is the current depicted in Fig. 3.10, \( J_1 \) is the functional to be compensated and \( T \) is the time period of \( v(t) \).

Thus, the reactor current is given as [5]

\[ i_L(\alpha, t) = \begin{cases} 
\frac{V}{\omega L} (\sin(\omega t) - \sin(\alpha)), & \alpha < \omega t < \pi - \alpha \\
\frac{V}{\omega L} (\sin(\omega t) + \sin(\alpha)), & \pi + \alpha < \omega t < 2\pi - \alpha.
\end{cases} \]  \hspace{1cm} (3.13)
3.3. DEVELOPMENT OF THE INDUSTRIAL SVC CONTROLLER

Figure 3.10: Optimal load compensation magnitudes.

Here, $\alpha$ is the thyristor firing angle, $L$ is the reactor inductance and $i_L(\alpha, t)$ is the reactor current. With (3.13) it is minimized (3.12) [5], where the next constrain was obtained

$$-\frac{4V}{\omega L} \left[ \int_{\alpha}^{\pi-\alpha} i(t) dt - \int_{\frac{2\pi-\alpha}{\omega}}^{\frac{\pi+\alpha}{\omega}} i(t) dt \right] = 0. \quad (3.14)$$

The constrain (3.14) was solved in real time and the whole information gathered in one-cycle was used in the following cycle to compensate for the reactive power [5]. This algorithm was extended in the paper for a three phase TCR connected in delta using the same ideas as in the single phase TCR [5].

3.2.3 Summary of the industrial SVC controllers

In this section two controllers were introduced, the first one uses phasors in order to perform the compensation. Thus this controller is really slow, since it will be acting cycles after the measurements were done [4]. Another thing that must be remarked from the variable susceptance controller is that it uses the classical power theory to achieve the compensation.

The second controller described here uses the notion of an optimization problem without restrictions to perform the compensation. The problem of this optimization problem is that it could affect the active power, since there is no restriction that it must be constant.

In the next section we develop a controller for an industrial SVC so it can compensate for the reactive power. Using the assumptions that the TCR is a variable reactor and that we are solving a current minimization problem with restrictions.

3.3 Development of the industrial SVC controller

The controller developed in this thesis was designed assuming that the voltages and currents of the test bench are sinusoidal and balanced, such as the working constrains given in Chapter 1. The objective of this controller is to compensates the current that produces no energy transfer between the electrical network and the load. So this controller must rely on the least-effort problem (2.49). Because of the assumed operation conditions, we will use the synchronous reference frame theory (2.33)-(2.36) to design the controller.

To obtain the desired condition, we will use the instantaneous powers (2.35)-(2.36),
\[ p = v_d i_d + v_q i_q, \]  
\[ q = v_d i_q - v_q i_d. \]  

(3.15)  
(3.16)

The quantities given in (3.15)-(3.16) were defined in Subsection 2.3.5. From that subsection we also know that under sinusoidal and balanced voltages, \( v_q = 0 \).

Thus, (3.15)-(3.16) turns into

\[ p = v_d i_d, \]  
\[ q = v_d i_q. \]  

(3.17)  
(3.18)

Since \( v_d \) is given by electrical grid, the quantity that will be our feedback signal is \( i_q \). Therefore we must achieve \( i_q = 0 \) so the instantaneous reactive power (3.18) becomes zero.

In order to, obtain a model that we can use to describe the current that produces no work. We need to decompose the current \( i_q \) into its components as it is depicted in Fig. 3.11.

![Figure 3.11: Current decomposition of \( i_q \).](image)

Thus,

\[ i_q = i_{q\text{Load}} + i_{q\text{TCR}} + i_{q\text{HF}}. \]  

(3.19)

Here, \( i_q \) is the total current to be compensated, \( i_{q\text{Load}} \) is the current in the \( q \) axis demanded by the load, \( i_{q\text{TCR}} \) is the current demanded by the TCR and \( i_{q\text{HF}} \) is the current drawn by the four FHF.

In this thesis, we assume that the TCR is a variable reactor, thus, we can neglect its harmonic components. Therefore, from (3.1), (3.9) and (2.85) we obtain the four FHFs and TCR currents expressed in the synchronous reference frame,

\[ i_{q\text{HF}} = \sum_{k=2}^{5} \sqrt{\frac{3}{2}} \omega V_{LN} C_{HF_{Yk}} \left( \frac{\omega_{0k}^2}{\omega_{0k}^2 - \omega} \right), \]  

(3.20)

\[ i_{q\text{TCR}} = -\frac{3}{\sqrt{2}} \frac{V_{LL}}{\pi \omega L} \frac{\sigma - \sin(\sigma)}{\sigma}. \]  

(3.21)
3.4 Discussion

The magnitude of the load $i_{qLoad}$ is not known. Thus, we would not define its value. Equations (3.19)-(3.21) describe the open-loop system depicted in Fig. 3.11 operating with sinusoidal and balanced voltages and currents. These equations are simplified by using a substitution variable $u$. This variable is implemented using a look-up table [27]. Thus, the open-loop system has the following form,

$$i_q = i_{qLoad} + i_{qTCR} + i_{qHF},$$

(3.22)

$$i_{qHF} = \sum_{k=2}^{5} \sqrt{\frac{3}{2}} \omega V_{LNC} C_{HFYk} \left( \frac{\omega^2_{0k}}{\omega^2_{0k} - \omega} \right),$$

(3.23)

$$i_{qTCR} = -3 \sqrt{\frac{2}{\pi \omega L}} u,$$

(3.24)

$$u = \sigma - \sin(\sigma),$$

(3.25)

$$\sigma = \pi - 2\alpha.$$  

(3.26)

With the equations (3.22)-(3.26) and using a PI-Controller. The system under closed-loop control takes the following form,

$$i_q = i_{qLoad} + i_{qTCR} + i_{qHF},$$

(3.27)

$$i_{qHF} = \sum_{k=2}^{5} \sqrt{\frac{3}{2}} \omega V_{LNC} C_{HFYk} \left( \frac{\omega^2_{0k}}{\omega^2_{0k} - \omega} \right),$$

(3.28)

$$i_{qTCR} = -3 \sqrt{\frac{2}{\pi \omega L}} u,$$

(3.29)

$$u = k_1(i_q - i_{qref}) + k_2 x_I,$$

(3.30)

$$\dot{x}_I = i_q - i_{qref}.$$  

(3.31)

Here, $u$ is a substitution variable implemented with a look-up table, $k_1$ and $k_2$ are the proportionality and integral gains of the PI-controller, $i_{qref}$ is the desired quadrature current and $x_I$ is the integral value of the error $i_q - i_{qref}$. In this thesis $i_{qref}$ will be taken as zero, because that is the desired $i_q$ value.

### 3.4 Discussion

In this chapter we developed the industrial SVC controller that operates under sinusoidal and balanced conditions. This controller is based on the least effort problem presented in Chapter 2. Therefore, it compensates the current that does not generate work. Also, the controller was designed assuming that the TCR could be abstracted as a variable reactor. Thus, we assume that the TCR is injecting a pure sinusoidal to the electrical grid.

In the next chapter we will implement the industrial SVC controller using equations (3.27)-(3.31).
Chapter 4

Experimental Setup

In this chapter, the implementation of the controller is described. In section 4.1 is specified how the hardware used to control the industrial SVC was designed. And section 4.2 describes how the controller using an RTOS was programmed.

4.1 Hardware development

The circuits developed to run the controller, are four:

- Power supply board
- Current sensors board
- Signal conditioning and DSP board
- Power interface board

The design criteria followed to develop these four boards will be described in the next subsections. The components and the layouts of these circuits are given in Appendix C and Appendix D respectively.

4.1.1 Power supply

This board supplies the required power to the other ones. Therefore, it provides 5V, 12V, -12V and the DC ground:

To achieve these specifications a linear power supply was designed. The topology is depicted in Fig 4.1.

Here, \( T_1 \) is a center-tapped transformer, \( Rec_1 \) and \( Rec_2 \) are the bridge rectifiers. And \( C_1, C_2 \) and \( C_3 \) are capacitors used as filters. To select these components we assumed that the whole linear regulators are consuming a maximum of 1 A of current.

With this assumption, the rectifier bridge TRS404L the selected. It can withstand 4 A of current and 400 V. The filter capacitors \( C_1, C_2 \) and \( C_3 \) were set at 2200 \( \mu \)F.

The only element left to be selected is the center-tapped transformer \( T_1 \). To select it an output voltage of 20 V is picked. Thus the apparent power of this transformer must be
at least 20 VA. The transformer fulfilling all the requirements is the 14A-30-20 from Signal Transformers [28]. The power supply developed for this application is depicted in Fig. 4.2.

4.1.2 Current sensors

To perform the selection of the current sensors, the current measurement of the four FHF's was performed. These currents were higher than 30 Arms. Therefore, this was taken as the selection constrain.

Then, using this constrain we selected the LA-55P, that is a Hall effect sensor from LEM. The advantages of this sensor is that it can measure DC and AC currents and it can measure a maximum current of 50 Arms [29]. The current sensors board is depicted in Fig. 4.3.
4.1. HARDWARE DEVELOPMENT

4.1.3 Signal conditioning

The voltage and current measurements must be conditioned to be fed into the DSP Analog to Digital Converter (ADC). These measurements uses almost the same topology based on operational amplifiers. Therefore, we present the computation needed to design these circuits. The topologies used to condition the current and voltage measurements are depicted in Fig. 4.4.

First the zener diode $D_z$ was selected. Since the ADC pins can withstand up to 3.3V [30]. Thus, the 1N4728 zener diode was chosen. This component will limit the input voltage to the ADC to 3.3 V.

Because the ADC from the DSP admits voltages in a range between zero an three volts. We must design the summing amplifier configuration of $OP_2$ with an input $V_i$ or $V_v$ to meet this criteria. The equation for the summing amplifier configuration is given by [31]

$$V_{out} = \left( \frac{R_3 + R_4}{R_3} \right) \left( \frac{V_{OP1} R_1}{R_1 + R_2} + \frac{V_r R_2}{R_1 + R_2} \right). \tag{4.1}$$

Here, $R_1, R_2, R_3$ and $R_4$ are the resistances depicted in Fig. 4.4. $V_{OP1}$ is the voltage coming from $OP_1$ in a voltage follower configuration with $V_i$ or $V_v$ as inputs. $V_r$ is the voltage.
The selected values for the parameters in (4.1) are given in Table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>50 kΩ</td>
</tr>
<tr>
<td>$R_2$</td>
<td>50 kΩ</td>
</tr>
<tr>
<td>$R_3$</td>
<td>100 kΩ</td>
</tr>
<tr>
<td>$R_4$</td>
<td>33 kΩ</td>
</tr>
<tr>
<td>$V_{OP1}$</td>
<td>[-2,2] V</td>
</tr>
<tr>
<td>$V_r$</td>
<td>2.5 V</td>
</tr>
</tbody>
</table>

Table 4.1: Selected values for the summing amplifier.

Using the numerical values from Table 4.1 in (4.1). We obtain

$$V_{out} = 1.33 \left( \frac{V_{OP1}}{2} + 1.25 \right).$$  

Therefore, the value $V_{out}$ will be given between 0.3325 V and 2.9925 V. To achieve the voltage $V_{OP1}$, we need to assure that the voltages $V_i$ and $V_v$ are in the range from -2 V to 2 V.

The voltage range in $V_i$ is achieved using a shunt resistor $R_c$. To select this shunt resistor we know from the LA-55P datasheet that the output current $i_s$ of this sensor is in the range from -70 mA to 70 mA. Thus, we select $R_c$ to be 29 Ω. With this resistance value we achieve the desired interval range.

For the voltage $V_v$ we select $R_v$ to be a 10 kΩ potentiometer. This potentiometer will be acting as a voltage divider from the potential transformer PT. The selected PT is the 44159 Isolation Transformer from Myrra.

The DSP and signal processing board is depicted in Fig. 4.5

### 4.1.4 Power interface

The power interface between the DSP and the power circuit has two parts. The contactor power interface and the thyristors firing circuit. First is described how the contactor power interface was designed and then the thyristor firing circuit.

**Contactor power interface**

The contactor power interface uses a relay to activate the contactor and a driver to isolate the DSP pins from the relay. The contactor power interface topology used in this thesis is depicted in 4.6.

The selected relay is the JS1A-12V-F from Panasonic. This relay uses 30 mA to be activated and can withstand a maximum voltage of 250 Vrms and a maximum current of 10 Arms [32].

To drive these relays the ULN2003A driver is used. Since the input of this component draws 0.93 mA from the DSP pin it can be driven without any other isolation circuit [33].
4.1. HARDWARE DEVELOPMENT

Thyristor firing circuit

The thyristor firing circuit consist in an AND logic gate and an optocoupler. The logic gate is used to enable the thyristor pulses that the DSP generates and as a overcurrent protection for the GPIO. The optocoupler is used to isolate the high voltage power device from the low voltage electronics. The topology for each thyristor firing circuit is depicted in Fig. 4.7.

The selected AND logic gate was the 74HCT08. This device was selected because it uses Complementary Metal-Oxide-Semiconductor technology. This technology draw from its inputs almost no current [31].
The resistance $R_d$ was computed assuming that the LED in the optocoupler was a short circuit and that a 15 mA is attained at 5 V. Therefore, the selected resistance value was 330 $\Omega$.

To select the resistance $R_{thy}$. We assume that the voltage in this resistance could be up to 300 V. And since the maximum current that the MOC3021M optocoupler can withstand is of 1 A. A resistance of 300 $\Omega$ at 5 W is selected.

The selected component are summarized in Table 4.2.

<table>
<thead>
<tr>
<th>Component</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optocoupler</td>
<td>MOC3021M</td>
</tr>
<tr>
<td>Logic gate</td>
<td>74HCT08</td>
</tr>
<tr>
<td>$R_d$</td>
<td>330 $\Omega$</td>
</tr>
<tr>
<td>$R_{thy}$</td>
<td>300 $\Omega$</td>
</tr>
</tbody>
</table>

Table 4.2: Selected values for the thyristor firing circuit.

The power interface board is depicted in Fig. 4.8.

### 4.2 Software development

The software development of the control algorithm uses an Real-Time Operating System (RTOS) to manage the threads of the system. The functions executed by the RTOS are:

- ADC measurement
- Voltage and current transformations
- Grid synchronization
- Firing pulse generation
- Closed-loop implementation

The RTOS used in this thesis and the functions used to make the experiment run are described below. The implementation code is given in Appendix E.
4.2. SOFTWARE DEVELOPMENT

4.2.1 Real-Time Operating System

The RTOS is in charge of the correct scheduling of the threads in a constrained time. Thus, with it we can assure that a certain thread will meet a deadline deterministically [34]. The RTOS used in this thesis is the TI-RTOS kernel. This RTOS was developed by Texas Instruments and it is available for the F28335 DSP.

The TI-RTOS kernel uses a priority-based preemptive scheduler [34]. This means that there will be different kinds of threads with its respective priority level and the scheduler will be running the highest priority thread every time. The threads used in this RTOS and its priority are depicted in Fig. 4.9.

The TI-RTOS kernel uses four kinds of threads, but in this thesis we will use just three.
The Hardware Interrupt (HWI), Software Interrupt (SWI) and clocks.

HWIs are used to process all the hardware interrupts of the DSP, such as the ADC and the Timer interrupts.

SWIs are functions that mimic the HWIs but with a lower priority. Thus, if a HWI is executed during the execution of a SWI it will preempt it and it will return to the SWI once the HWI has been finished.

Clocks are SWIs that mimic a timer interrupt. But it is implemented by software instead of a hardware implementation.

The program used to control the SVC is running a clock thread at 12 kHz where it will ask the ADC to prepare a value. In each timestamp, the program will read the ADC with the HWI posted by the clock thread. Then, it will post a SWI where it will calculate the voltage and current transformations, the angle of the grid, the firing angle and the firing pulse generator. When the fire pulse generator is activated it will post a timer HWI to turn off the pin. This pulse will last for 100 $\mu$S. A timestamp of the implementation is depicted in Fig. 4.10.

![Figure 4.10: Industrial SVC controller implementation.](image)

The dashed lines in Fig. 4.10 are the timer HWI post done by the SWI and the return of the timer HWI. This HWI will preempt the SWI and return to finish the computation of the SWI.

The flow diagram of the scheduled interrupts by the RTOS are depicted in Fig. 4.11. In the next subsections each calculation performed at the SWI will be explained.

### 4.2.2 Voltage and current transformations

Once the ADC values are stored. The voltages are transformed into the $\alpha$-$\beta$ coordinates. This is done using (2.29).

Then, with these quantities, the grid angle is calculated,

$$\theta = \arctan \left( \frac{v_\beta}{v_\alpha} \right).$$  (4.3)
4.2. SOFTWARE DEVELOPMENT

The function used to get the angle is the atan2 function and it takes 49 CPU cycles to be performed [35].

With the angle given in (4.3) the transformation of the currents into the \( d - q \) coordinate frame using (2.34) is performed. The computation of the sines and cosines is done using the sincos function. Since this function uses just 44 CPU cycles [35]. Instead of the 37 and 38 CPU cycles that are needed to compute the sine and cosine functions in a separate way [35].

The grid angle in (4.3) is the angle of the line to neutral voltages. Thus in the next section we will explain how we transformed the line to neutral angle to line to line angles.

4.2.3 Synchronization

Since the measured voltage is from line to neutral and it is assumed that they are balanced. Then, we can apply an offset to \( \theta \) to obtain the voltages from line to line. Then, the line to line angles are given by,

\[
\theta_{ab} = \theta + \frac{\pi}{6}, \quad \theta_{bc} = \theta - \frac{\pi}{2}, \quad \theta_{ca} = \theta + \frac{5\pi}{6}.
\]
With the transformed angles given in (4.4)-(4.6) we perform the firing pulse generation that is explained in the next subsection.

### 4.2.4 Firing pulse generation

To generate the firing pulses, we implemented a Finite State Machine (FSM) for each phase in the TCR. Thus, this FSM assures that the pulse will be fired just once in every phase half cycle. The FSM for each phase is depicted in Fig. 4.12.

![Firing pulse finite state machine](image)

Here, Thy\(_{xy}^+\) and Thy\(_{xy}^-\) are the thyristors in the positive and negative half cycle of the phase \(xy\). The angle \(\theta_{xy}\) is calculated in the synchronization stage for the phase \(xy\). Finally, \(\alpha\) is the firing angle of the TCR.

The FSMS for the TCR phases work as follows. Each FSM will be checking its angle \(\theta_{xy}\) and once the condition under which the TCR phase angle is bigger than the firing angle is met the DSP output pin is turned on and the FSM changes the state, so the pin is not toggling. To turn off that pin, a timer HWI is activated before the FSM changes the state. This timer HWI checks the state of the FSM and with that information it turn off the right pin.

The thyristor firing order is given in Table 4.3.

### 4.2.5 Closed-loop implementation

The closed-loop implementation was done using the system given by (3.27)-(3.31). The signal flow of this implementation is depicted in Fig. 4.13.

In order to implement the PI-controller and the look-up table, the voltage and current transformations and the firing pulse generator are needed. The restriction imposed to this controller is that it can change the firing angle \(\alpha\) just once in a cycle. This restriction was imposed because the TCR is operating under balanced conditions.
4.3 Discussion

In this chapter we describe how the hardware to implement the controller developed in Chapter 3 was designed. Then, we used an RTOS to schedule the controller logic. Finally, we join the

<table>
<thead>
<tr>
<th>Thyristor Number</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Thy$_{ab}$ +</td>
<td>1</td>
</tr>
<tr>
<td>Thy$_{ab}$ -</td>
<td>4</td>
</tr>
<tr>
<td>Thy$_{bc}$ +</td>
<td>3</td>
</tr>
<tr>
<td>Thy$_{bc}$ -</td>
<td>6</td>
</tr>
<tr>
<td>Thy$_{ca}$ +</td>
<td>5</td>
</tr>
<tr>
<td>Thy$_{ca}$ -</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4.3: Thyristor firing order.

The PI-controller code was obtained with the code generator from PSIM 11. This controller is implemented with a backward Euler algorithm for the integration, then the program saturates the integral to specified values. Finally, the saturated integral is summed to the proportional of the error and then this sum in saturated to the same values. In this thesis the PI-controller restrict the values of $u$ to lie between 0.023596 and 2.117992. This values of $u$ correspond to an $\alpha$ between $15^\circ$ and $75^\circ$. The restriction in $\alpha$ was imposed because the TCR reactor was designed to operate under those conditions. The PI-controller code is given in Appendix E.

Also the look-up table was obtained using PSIM 11. The look-up table uses ten values to obtain the interpolated values of $\alpha$ for a certain $u$. These values are given in Table 4.4.

4.3 Discussion

Figure 4.13: Closed-loop implementation.
Table 4.4: Implemented look-up table.

<table>
<thead>
<tr>
<th>$u$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.570795</td>
</tr>
<tr>
<td>0.007043196</td>
<td>1.396262222</td>
</tr>
<tr>
<td>0.05534192</td>
<td>1.221729444</td>
</tr>
<tr>
<td>0.181170378</td>
<td>1.047196667</td>
</tr>
<tr>
<td>0.411454213</td>
<td>0.872663889</td>
</tr>
<tr>
<td>0.76052023</td>
<td>0.698131111</td>
</tr>
<tr>
<td>1.228368372</td>
<td>0.523598333</td>
</tr>
<tr>
<td>1.800671731</td>
<td>0.349065556</td>
</tr>
<tr>
<td>2.450504578</td>
<td>0.174532778</td>
</tr>
<tr>
<td>3.14159</td>
<td>0</td>
</tr>
</tbody>
</table>

hardware and software implementations to deploy the system. The results of the experiments are presented in the next chapter.
Chapter 5

Results

In this chapter we present the results of the experiments. The measurements of the experiments were done at the Point of Common Coupling (PCC) and they are depicted in Fig. 5.1.

Here, $v_a(t)$, $v_b(t)$ and $v_c(t)$ are the voltages of the phases $a$, $b$ and $c$. The line currents of the phases $a$, $b$ and $c$ are $i_a(t)$, $i_b(t)$ and $i_c(t)$ respectively. These voltages and currents are depicted in different colors because we will use these colors to identify each phase measurement in the following sections. The simulation results are given in Appendix F.

5.1 Open-loop results

In this section we present the results of the open-loop experiments. We performed five measurements. The first one was to measure the four FHFs voltages and currents. And the other four were the voltages and currents measurements of the TCR at various firing angles.

These measurements were taken with the AEMC 8335. This device captures 256 samples per cycle. The captured waveforms are shown below.

5.1.1 Fixed harmonics filters

In this experiment we connected the four FHFs and measure the voltages and currents at the PCC. The resulting voltages and currents are depicted in Fig. 5.2.
From Fig. 5.2 we can see that the currents are leading the voltages by $90°$. The RMS and THD values of these voltages and currents are given Table 5.1.

<table>
<thead>
<tr>
<th>Voltage</th>
<th>RMS</th>
<th>THD</th>
<th>Current</th>
<th>RMS</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_a(t)$</td>
<td>147.4 Vrms</td>
<td>0.4%</td>
<td>$i_a(t)$</td>
<td>31.41 Arms</td>
<td>2.2%</td>
</tr>
<tr>
<td>$v_b(t)$</td>
<td>146.1 Vrms</td>
<td>0.4%</td>
<td>$i_b(t)$</td>
<td>31.34 Arms</td>
<td>1.9%</td>
</tr>
<tr>
<td>$v_c(t)$</td>
<td>146.2 Vrms</td>
<td>0.3%</td>
<td>$i_c(t)$</td>
<td>31.27 Arms</td>
<td>2.3%</td>
</tr>
</tbody>
</table>

Table 5.1: Computed RMS and THD values for each parameter.

5.1.2 TCR

These experiments were done to prove the firing pulse generator system using the FSM described in subsection 4.2.4. We set four firing angles in order to see the response of the TCR with the one given in theory. These firing angles and the resulting voltages and currents are given below.
5.1. OPEN-LOOP RESULTS

TCR $\alpha = 30^\circ$

This experiment was done to see how the RMS and THD values of the voltages and currents consumed by the TCR were behaving at $\alpha = 30^\circ$. The obtained plots are depicted in Fig. 5.3.

![Voltage and Current Plots](image)

From Fig. 5.3 we can see that the current is lagging the voltage by 90°. The notches in the voltage are because of the thyristor firing. The RMS and THD values of the voltages and currents depicted in Fig. 5.3 are given in Table 5.2.

<table>
<thead>
<tr>
<th>Voltage</th>
<th>RMS</th>
<th>THD</th>
<th>Current</th>
<th>RMS</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_a(t)$</td>
<td>119.2 Vrms</td>
<td>2.8%</td>
<td>$i_a(t)$</td>
<td>41.53 Arms</td>
<td>2.6%</td>
</tr>
<tr>
<td>$v_b(t)$</td>
<td>119 Vrms</td>
<td>2.8%</td>
<td>$i_b(t)$</td>
<td>41.58 Arms</td>
<td>2.6%</td>
</tr>
<tr>
<td>$v_c(t)$</td>
<td>119 Vrms</td>
<td>2.8%</td>
<td>$i_c(t)$</td>
<td>41.68 Arms</td>
<td>2.1%</td>
</tr>
</tbody>
</table>

Table 5.2: Computed RMS and THD values for each parameter.
**TCR $\alpha = 45^\circ$**

We increased the value of the firing angle to see the increment of the THD in the current and how will be behaving the voltage at the PCC when the current is that distorted. In Fig. 5.4 are depicted the voltages and currents at $\alpha = 45^\circ$.

![Graph showing voltages and currents]

Figure 5.4: Measurements at the PCC of the TCR with an $\alpha = 45^\circ$.

From Fig. 5.4 we can see that the demanded current by the TCR is still so high that it can change the voltage at the PCC. The RMS and THD values of the voltage and currents at $\alpha = 45^\circ$ are given in Table 5.3.

<table>
<thead>
<tr>
<th>Voltage</th>
<th>RMS</th>
<th>THD</th>
<th>Current</th>
<th>RMS</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_a(t)$</td>
<td>128.1 Vrms</td>
<td>4.9%</td>
<td>$i_a(t)$</td>
<td>18.81 Arms</td>
<td>19%</td>
</tr>
<tr>
<td>$v_b(t)$</td>
<td>127.8 Vrms</td>
<td>5.2%</td>
<td>$i_b(t)$</td>
<td>18.7 Arms</td>
<td>19.7%</td>
</tr>
<tr>
<td>$v_c(t)$</td>
<td>127.5 Vrms</td>
<td>4.9%</td>
<td>$i_c(t)$</td>
<td>18.61 Arms</td>
<td>20.3%</td>
</tr>
</tbody>
</table>

Table 5.3: Computed RMS and THD values for each parameter.
5.1. OPEN-LOOP RESULTS

TCR $\alpha = 60^\circ$

In this experiment we used a firing angle of $\alpha = 60^\circ$. The demanded current by the TCR diminish at this firing angle. This is depicted in Fig. 5.5.

![Voltage and Current Waveforms](image)

Figure 5.5: Measurements at the PCC of the TCR with an $\alpha = 60^\circ$.

Here, because the current demanded by the TCR is lower, the voltage at the PCC is less distorted. But the THD of the current is higher. The measured RMS and THD values of the voltage and currents at the PCC are given in Table 5.4.

<table>
<thead>
<tr>
<th>Voltage</th>
<th>RMS</th>
<th>THD</th>
<th>Current</th>
<th>RMS</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_a(t)$</td>
<td>133.6 Vrms</td>
<td>3.9%</td>
<td>$i_a(t)$</td>
<td>5.09 Arms</td>
<td>56.3%</td>
</tr>
<tr>
<td>$v_b(t)$</td>
<td>133 Vrms</td>
<td>4.1%</td>
<td>$i_b(t)$</td>
<td>5.21 Arms</td>
<td>56.8%</td>
</tr>
<tr>
<td>$v_c(t)$</td>
<td>132.9 Vrms</td>
<td>3.6%</td>
<td>$i_c(t)$</td>
<td>5.17 Arms</td>
<td>57.4%</td>
</tr>
</tbody>
</table>

Table 5.4: Computed RMS and THD values for each parameter.
TCR $\alpha = 75^\circ$

In order to see the behaviour of the TCR at the maximum firing angle allowed by the TCR designers we set $\alpha = 75^\circ$. The voltages and currents at the PCC are depicted in Fig. 5.6.

![Figure 5.6: Measurements at the PCC of the TCR with an $\alpha = 75^\circ$.](image)

In Fig. 5.6 the demanded current diminished. Therefore, the impact of the thyristor firing in the voltage is lower. The RMS and THD values of the voltages and currents at the PCC are given in Table 5.5.

<table>
<thead>
<tr>
<th>Voltage</th>
<th>RMS</th>
<th>THD</th>
<th>Current</th>
<th>RMS</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_a(t)$</td>
<td>134.9 Vrms</td>
<td>1.9%</td>
<td>$i_a(t)$</td>
<td>1.09 Arms</td>
<td>113%</td>
</tr>
<tr>
<td>$v_b(t)$</td>
<td>134.3 Vrms</td>
<td>1.8%</td>
<td>$i_b(t)$</td>
<td>1.08 Arms</td>
<td>125.7%</td>
</tr>
<tr>
<td>$v_c(t)$</td>
<td>134.3 Vrms</td>
<td>1.9%</td>
<td>$i_c(t)$</td>
<td>1.02 Arms</td>
<td>118.2%</td>
</tr>
</tbody>
</table>

Table 5.5: Computed RMS and THD values for each parameter.
5.2 Closed-loop results

In order to evaluate the performance of the industrial SVC controller developed in this thesis, an implementation that fulfills the conditions under which this controller works was set up. For this experiment the used loads are an induction motor loaded with a DC generator that feeds a resistor bank and a capacitor bank. The induction motor is used to show that the controller can compensate a load that is commonly used in industry and the capacitor bank is used to perturbate the system in order to see how the controller behaves under randomly applied loads.

In this experiment, the DC generator is set to an output voltage of 90 V and it is feeding a resistor bank of 3 Ω. The capacitor bank was set up using a variable capacitor from De-Lorenzo connected in delta and a wye connected fixed capacitor bank from Westinghouse to an autotransformer from General Radio company.

The three phase power in steady state demanded by the loads are given in Table 5.6.

<table>
<thead>
<tr>
<th>Load</th>
<th>Active Power</th>
<th>Reactive Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Induction motor</td>
<td>2.89 kW</td>
<td>3.52 kVAR</td>
</tr>
<tr>
<td>Capacitor bank</td>
<td>0.225 kW</td>
<td>4.56 kVAR</td>
</tr>
</tbody>
</table>

Table 5.6: Demanded power by the loads at 220 $V_{LL}$.

This experiment will be activating each power device in a series of steps in order to see the transients of the devices and how the controller react to those cases. The turn on and off sequence of this experiment is given in Table 5.7.

<table>
<thead>
<tr>
<th>Load</th>
<th>Turning</th>
<th>Time interval [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Induction motor and capacitor bank</td>
<td>On</td>
<td>15</td>
</tr>
<tr>
<td>TCR and $HF_2$</td>
<td>On</td>
<td>2.5</td>
</tr>
<tr>
<td>$HF_3$</td>
<td>On</td>
<td>2.5</td>
</tr>
<tr>
<td>$HF_4$</td>
<td>On</td>
<td>2.5</td>
</tr>
<tr>
<td>$HF_5$</td>
<td>On</td>
<td>300</td>
</tr>
<tr>
<td>$HF_6$</td>
<td>Off</td>
<td>2.5</td>
</tr>
<tr>
<td>$HF_4$</td>
<td>Off</td>
<td>2.5</td>
</tr>
<tr>
<td>$HF_3$</td>
<td>Off</td>
<td>2.5</td>
</tr>
<tr>
<td>TCR and $HF_2$</td>
<td>Off</td>
<td>2.5</td>
</tr>
<tr>
<td>Induction motor and capacitor bank</td>
<td>Off</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.7: Turn on and off sequence.

During the 15 s between the power on of the induction motor and capacitor bank breakers and the TCR and $HF_2$ breakers the resistor bank was configured and the capacitor bank was turned on once the inrush current of the motor diminished. In between the powering on and off of the $HF_5$ breaker the capacitor bank was switched on and off to evaluate the controller performance against sudden changes in the load.
CHAPTER 5. RESULTS

The closed-loop measurements were taken with the Elspec G4500 and three SOA-9045-3001. Since the SOA-9045-3001 has a measurement range between 90A and 3000A the measured cable was rolled ten times to multiply the current by ten, then this was corrected at the configuration of the Elspec. Finally, the Elspec G4500 was configured to store 512 voltage samples and 128 current samples per cycle.

5.2.1 Complete data set

The RMS current values of the whole experiment are depicted in Fig. 5.7.

The overshoots that are present in Fig. 5.7 between 48 s and 250 s are because of the connection and disconnection of the capacitor bank. In order to have a better understanding of Fig. 5.7, we will zoom this plot. In the next subsections we will do a zoom to the power up of the motor and capacitor bank, the turning on sequence of the industrial SVC and the evaluation of the industrial SVC controller using the capacitor bank as a perturbing load.
5.2. CLOSED-LOOP RESULTS

5.2.2 Motor and capacitor bank

In this part of the experiment, the motor was powered on, then it was loaded with a DC generator that was feeding a 3 \( \Omega \) resistor bank with 90V. At the end of this part of the experiment the capacitor bank was turned on. In Fig. 5.8 are depicted the RMS currents and the \( i_q(t) \) current when the motor and capacitor bank breaker is power on.

![Graph showing RMS currents and quadrature current with zones labeled](image)

Figure 5.8: Load RMS value and \( i_q(t) \) current.

In Fig. 5.8 were highlighted three zones, \( Z_1 \), \( Z_2 \) and \( Z_3 \). Zone \( Z_1 \) depicts how the RMS currents and \( i_q(t) \) current behave when the induction motor is turned on without load. Zone \( Z_2 \) depicts how the motor behaves in steady state without load. The resistor bank is turned on in zone \( Z_3 \). Finally the zone that is not highlighted depicts the turn on of the capacitor bank.
## 5.2.3 Industrial SVC turn on sequence

The second part of the experiment is the turn on sequence of the industrial SVC. Here, the controlled TCR and the four FHF s are powered on. The turn on sequence of the industrial SVC is depicted in Fig. 5.9. This sequence begins with the power on of the TCR and the second harmonic filter, then every 2.5 seconds a FHF will be power on as it is given in Table 5.7.

![RMS Current and Quadrature Current](image)

Figure 5.9: RMS currents and $i_q(t)$ current when the industrial SVC is turning on.

In these figures are four overshoots, every one of them is the turning on instant of the FHF s and the first overshoot includes the TCR ignition. These are due to the transient currents of the FHF s and the transient behavior of the industrial SVC controller.
5.2.4 Evaluation of the industrial SVC controller

After the fifth harmonic filter is powered. The capacitor bank is being turned on and off randomly to evaluate the industrial SVC controller performance. In Fig. 5.10 are depicted the RMS current values and the $i_q(t)$ current when the capacitor bank is being turned off and on.

![Graph showing RMS currents and quadrature current](image)

Figure 5.10: RMS Currents and the $i_q(t)$ current when the capacitor bank is switched off and on.

The controller is evaluated in the switching off and on of the capacitor bank. When the capacitor bank is switched off there is a small transient in the current and this is due to the transient behaviour of the controller. The other case is when the capacitor bank is turned on and there is an overshoot in the RMS value of the currents because of the transient phenomena in the capacitor bank and the TCR controller.

The transient currents at the PCC and at the TCR when the capacitor bank is being turned off and on are described below.

**Capacitor bank turning OFF currents**

The transient current when the capacitor bank is switched off corresponds to the transitory behavior of the industrial SVC controller. Since the capacitor bank produced no transient
when it is switched off, it just stays at its last applied voltage.

The transient current at the PCC when the capacitor bank is switched off are depicted in Fig. 5.11.

![Figure 5.11: Currents when the capacitor bank is switched off at the PCC.](image)

The transient currents at the TCR when the capacitor bank is switched off are depicted in Fig. 5.11.

![Figure 5.12: Currents when the capacitor bank is switched off at the TCR.](image)

The measurements of the TCR currents depicted in Fig. 5.12 were obtained with an MDO4024C Tektronix oscilloscope and three TCP0150 Tektronix current probes.
Capacitor bank turning ON currents

The transient in the current at the PCC when the capacitor bank is turned on corresponds to the connection of the capacitor bank and the industrial SVC controller.

The currents at the PCC when the capacitor bank is turned on are depicted in Fig. 5.13.

![Figure 5.13: Currents when the capacitor bank is switched on at the PCC.](image)

The measurements of the TCR currents depicted in Fig. 5.14 were obtained with an MDO4024C Tektronix oscilloscope and three TCP0150 Tektronix current probes.

![Figure 5.14: Currents when the capacitor bank is switched on at the TCR.](image)
Chapter 6

Conclusion

In order to have a better understanding of the industrial SVC working principle. This thesis was divided in three main parts,

- Power theory,
- Industrial SVC controller development,
- Industrial SVC controller implementation.

Each one of them was studied in a separated chapter, and we will summarize them below.

In Chapter 2 we study the history of the power theory and the major theories that try to understand the phenomena. After that we develop a framework to have a physical understanding of the power phenomena in AC circuits. This was done using the least-effort problem. Using this framework we obtained the generalization of the reactive power for an $n$ phase system with non-sinusoidal voltages and currents. Also, since the least-effort problem solves for the minimal current that achieves the same energy transfer in the time interval $[0, T]$. We extended the power theory to DC circuits and non-periodic functions because they also belong to the space where the least-effort problem holds.

In Chapter 3 we develop the industrial SVC controller assuming that the voltages and currents are sinusoidal and balanced. Thus, we used the synchronous reference frame transformation to design the controller. With that theory we used the $i_q$ as the feedback signal. And it was fed to a PI-controller with a look-up table to attain $i_q = 0$. Thus, the controller solves the least-effort problem when the system is a three phase system with sinusoidal and balanced voltages and currents.

In Chapter 4 we developed the hardware to measure the desired signals from the power system and the power interface to turn on the desired power devices with the DSP. Then, we used an RTOS to implement the industrial SVC controller logic.

Finally, the results of the implementation are presented in Chapter 5. Here, we present the voltages and currents for the open-loop case and the RMS values and the $i_q$ current of the closed-loop implementations. In the open-loop implementation of the TCR and the four FHFs everything was working as expected. In the closed-loop implementation, the TCR was compensating the $i_q$ current, but the value $i_q = 0$ was not achieved since the voltages and currents were not perfectly sinusoidal as it was assumed in the controller development.
6.1 Future work

The current implementation of the industrial SVC controller assumes that the voltages and currents are balanced and sinusoidal. The implementation of the hardware uses op-amps for the signal conditioning circuit. The problem of this implementation is that this circuit injects noise into the signal. Also, there is just one DSP doing the calculation. Thus, if it fails the system will collapse.

Therefore, the future work to do is the following:

- Design an industrial SVC controller that minimizes the current that produces no energy transfer between the load and the distribution network when the currents and voltages are distorted. This could be done using an optimal control approach [22, 23] because the least-effort problem is an optimization problem with restrictions [23].

- To minimize the noise produced by the op-amps, we would change them for an opto-coupled implementation to have an optical isolation from the measured source.

- Use more DSPs in parallel to assure that the system will not collapse if one of them stop functioning.

- Prove the stability of the close-loop system.

- Prove the feasibility of the real-time implementation.

- Scale the power system to a medium voltage implementation.
Appendix A

Hilbert space

The Hilbert space is the generalization of the Euclidean space when the number of elements needed to describe another element in that set can be infinite [22, 23, 36]. In this generalization we can define a length of the elements belonging to this set. Also, we can give a definition of the angle between two elements and the most important part, with this space we can define projections of one element into another one [22, 23, 36]. This section describes the properties of the Hilbert space used to have a better understanding of the power theory.

A.1 Vector spaces

In order to give a definition of the Hilbert space, first we must define a vector space. A vector space $X$ is a set of elements called vectors, where two operations are defined. The first operation is vector addition that associate two vectors $x, y \in X$ a vector $x + y \in X$. The second operation is scalar multiplication [22], where $\forall x \in X$ and a scalar $\alpha$, it is associated a vector $\alpha x \in X [22]$. These operations are assumed to satisfy the following axioms [22]:

1. $x + y = y + x$.
2. $(x + y) + z = x + (y + z)$.
3. $(\alpha + \beta)x = \alpha x + \beta x$.
4. $(\alpha \beta)x = \alpha (\beta x)$.
5. $x + 0 = x \forall x \in X$.
6. $\alpha(x + y) = \alpha x + \alpha y$.
7. $0x = 0$.

In these axioms, the vector $z \in X$ and $0$ is called the null vector [22]. The first four axioms are known as the commutative law, associative law for vector sums, distributive law and the associative law for scalar multiplication, respectively [22].

With the vector addition and scalar multiplication specified, we can define a linear combination of the vectors $x_1, x_2, x_3, \ldots, x_n$ in a vector space as (A.1) [22],
\[
\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \cdots + \alpha_n x_n \in X. \tag{A.1}
\]

Thus, with the linear combination we can span the whole space \( X \) with a given number of vectors. Where these vectors will be called the basis of the vector space and the number of vectors needed \( n \) will be called the dimension of the vector space \([22, 23, 36]\). With the definition of linear combination, we can give the definition of a subspace of a vector space.

**Definition:** A subset \( M \) of a vector space \( X \) is a subspace of \( X \) if every linear combination of the form \( \alpha_1 x_1 + \alpha_2 x_2 \) is in \( M \).

### A.2 Norm and inner product

The Hilbert space is a vector space where we add an operation that we will call it the inner product \([22, 23, 36]\). This operation is defined between two vectors \( x, y \in X \) as the ordered pair \( \langle x, y \rangle \), where the result of this operation is a scalar \([22, 23, 36]\). This operation satisfies the following axioms \([22, 23, 36]\):

1. \( \langle x, y \rangle = \langle y, x \rangle \).
2. \( \langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle \).
3. \( \langle \alpha x, y \rangle = \alpha \langle y, x \rangle \).
4. \( \langle x, x \rangle \geq 0 \) and \( \langle x, x \rangle = 0 \) iff \( x = 0 \).

With these axioms, we define the norm of the vector \( x \) by \( \langle A.2 \rangle \)[22, 23, 36],

\[
\| x \| = \sqrt{\langle x, x \rangle} \tag{A.2}
\]

The norm \( \langle A.2 \rangle \) is a generalization of the notion of length in a Hilbert space \([22, 23, 36]\). And it is used in one of the most used inequalities given by Hilbert spaces, the Cauchy-Schwarz inequality \( [22] \). This inequality is the written as,

\[
\langle x, y \rangle \leq \| x \| \| y \|. \tag{A.3}
\]

The Cauchy-Schwarz inequality is converted into an equality iff \( x = \alpha y \) or \( y = 0 \) \( [22] \). When the inner product \( \langle x, y \rangle \) is a real number, we define the angle \( \phi \) between \( x \) and \( y \) as \( [23] \),

\[
\cos \phi = \frac{\langle x, y \rangle}{\| x \| \| y \|}. \tag{A.4}
\]

Thus, from \( \langle A.4 \rangle \) we can say that the vectors \( x \) and \( y \) are orthogonal if \( \langle x, y \rangle = 0 \), and we will write it as \( x \perp y \) \( [22] \). With this geometrical interpretation we will give the definition of projections in a Hilbert space.
A.3 Projection theorems

The main advantage of Hilbert spaces are the projections of vectors. These theorems are used to solve norm optimization problems [22]. The theorems used to compute projections are the following [22, 23],

**Theorem 1** Let $X$ be a Hilbert space and $M$ a subspace of $X$, and $x$ is an arbitrary vector in $X$. If there exist a vector $m_0$ such that $\|x - m_0\| \leq \|x - m\| \forall m \in M$, then $m_0$ is unique [22, 23]. The necessary and sufficient condition that $m_0 \in M$ be a unique minimizing vector in $M$ is that the error vector $x - m_0$ be orthogonal to $M$ [22, 23].

The proof of Theorem 1 is given in [22], where it is proved that if $m_0$ is a minimizing vector then $x - m_0$ is orthogonal to $M$. In order to have a geometrical insight to this theorem, a three dimensional version of it is depicted in Fig. A.1.

![Figure A.1: Projection in a three dimensional space.](image)

Theorem 1 has the restriction that the projection must be done to a subspace of $X$. Thus, it is extended for the cases when the norm minimization will be done in a shifted subspace in Theorem 2. We will call that shifted subspace a hyperplane [23].

**Theorem 2** Let $X$ be a Hilbert space with a subspace $M$. Let $V$ be the hyperplane $x + M$, where $x$ is a fixed vector belonging to $X$. Then, there exists a vector $x_0$ in $V$ that is unique and of minimum norm. Also, $x_0$ is orthogonal to $M$ [22].

In order to prove Theorem 2, we must translate the hyperplane $V$ to the origin so it becomes $M$ and then apply the projection theorem to obtain $m_0$ [22]. Thus, the minimum norm vector from Theorem 2 is $x_0 = x - m_0$ [22]. A geometrical interpretation of this proof is depicted in Fig. A.2.
A.4 Least-effort problem by orthogonal projection

The least-effort problem, is an optimization problem that looks for the closest vector to the origin, where this vector is constrained by inner products [22, 23]. This can be achieved if the problem follows this theorem [22, 23].

Theorem 3  Let the set of linearly independent vectors \( \{x_1, x_2, \ldots, x_k\} \) belonging to a Hilbert space \( X \) span a subspace \( M \) of \( X \). Then, among all vectors \( x \in X \) that satisfy,

\[
\langle x, x_p \rangle = c_p \quad p = 1, \ldots, k. \tag{A.5}
\]

And letting \( x_0 \) be the minimum norm vector. Then

\[
x_0 = \sum_{p=1}^{k} \beta_p x_p. \tag{A.6}
\]

Where the constants \( \beta_1, \beta_2, \ldots, \beta_k \) are obtained by solving the system of equations (A.7), known as the Gram equations [23],

\[
\frac{\langle x_1, x_1 \rangle}{\langle x_1, x_2 \rangle} \quad \frac{\langle x_2, x_1 \rangle}{\langle x_2, x_2 \rangle} \quad \ldots \quad \frac{\langle x_k, x_1 \rangle}{\langle x_k, x_2 \rangle}

\begin{pmatrix}
\vdots & \vdots & \ddots & \vdots \\
\langle x_1, x_k \rangle & \langle x_2, x_k \rangle & \ldots & \langle x_k, x_k \rangle
\end{pmatrix}

\begin{pmatrix}
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_k
\end{pmatrix}

= \begin{pmatrix}
c_1 \\
c_2 \\
\vdots \\
c_k
\end{pmatrix}. \tag{A.7}
\]

The proof of Theorem 3 is given in [22, 23] and uses ideas from Theorem 2. Here we will describe the followed steps. Since, the intersection of the whole hyperplanes defined by (A.5) result in the hyperplane \( V \) from Theorem 2 [23]. Now to find the minimizing vector \( x_0 \) we must find a orthogonal vector to \( V \). To obtain that orthogonal vector, we know that the subspace \( M \) is orthogonal to \( V \) [22, 23]. Thus, we express the minimum norm vector \( x_0 \) as a linear combination of the vectors \( \{x_1, x_2, \ldots, x_k\} \) (A.6). Finally, we obtain the intersection between \( M \) and \( V \) with the substitution of (A.6) into (A.5) where we obtain the Gram equations (A.7) [22, 23].
A geometrical interpretation of Theorem 3 is depicted in Fig. A.3.

Theorem 3 can be applied to every Hilbert space [22, 36]. Thus, in subsections A.5, A.6 and A.7 we will define the Hilbert spaces that are used in this thesis. These Hilbert spaces depend on the functions used to represent voltages and currents.

### A.5 Real $L^2_n[a, b]$ space

The real $L^2_n[a, b]$ space is the set of all real vectors valued functions of dimension $n$, that are square-integrable in the interval $[a, b]$ [37]. Thus, $X(t) \in L^2_n[a, b]$ holds if (A.8) is true,

$$\int_a^b X(t)^\top X(t) dt < \infty.$$ (A.8)

The inner product for this Hilbert space is defined as follows [37],

$$\langle X(t), Y(t) \rangle \triangleq \frac{1}{b - a} \int_a^b X(t)^\top Y(t) dt.$$ (A.9)

Here, the vector valued function $Y(t)$ also belongs to $L^2_n[a, b]$. From (A.9) we can derive the induced norm of $X(t)$,

$$\|X(t)\| \triangleq \sqrt{\frac{1}{b - a} \int_a^b X(t)^\top X(t) dt}.$$ (A.10)

With the definition of the inner product (A.9) and norm (A.10). We can now write the Cauchy-Schwarz inequality for this Hilbert space,
\[
\left( \frac{1}{b-a} \int_a^b X(t)^\top Y(t)dt \right)^2 \leq \frac{1}{(b-a)^2} \left( \int_a^b X(t)^\top X(t)dt \right) \left( \int_a^b Y(t)^\top Y(t)dt \right). \tag{A.11}
\]

The inequality (A.11) becomes an equality if the following term is added to the left hand side of (A.11),
\[
\frac{1}{2(b-a)^2} \int_a^b \int_a^b \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} (y_r(t)x_s(\tau) - y_s(\tau)x_r(t))^2 dtd\tau. \tag{A.12}
\]

The derivation of the term (A.12) is given at the subsection A.5.1. Thus, the equality has the form,
\[
\left( \frac{1}{b-a} \int_a^b X(t)^\top Y(t)dt \right)^2 + \frac{1}{2(b-a)^2} \int_a^b \int_a^b \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} (y_r(t)x_s(\tau) - y_s(\tau)x_r(t))^2 dtd\tau
= \frac{1}{(b-a)^2} \left( \int_a^b X(t)^\top X(t)dt \right) \left( \int_a^b Y(t)^\top Y(t)dt \right). \tag{A.13}
\]

It must be noted that (A.12) vanishes if \(X(t)\) and \(Y(t)\) are proportional or one of them is the null vector.

Finally, we will define the basis for this space. The used basis is the real Fourier basis, so every vector belonging to \(L^2[a, b]\) can be obtained as a linear combination of this basis. Thus,
\[
X(t) = \begin{pmatrix} a_{01} + \sum_{k=1}^{\infty} a_{k1} \cos(2\pi f kt) + \sum_{k=1}^{\infty} b_{k1} \sin(2\pi f kt) \\ a_{02} + \sum_{k=1}^{\infty} a_{k2} \cos(2\pi f kt) + \sum_{k=1}^{\infty} b_{k2} \sin(2\pi f kt) \\ \vdots \\ a_{0n} + \sum_{k=1}^{\infty} a_{kn} \cos(2\pi f kt) + \sum_{k=1}^{\infty} b_{kn} \sin(2\pi f kt) \end{pmatrix}. \tag{A.14}
\]

Here, \(f\) is the fundamental frequency and \(k\) is the harmonic number.

A.5.1 Derivation of the real \(L^2[a, b]\) space equation

The quantity given by (A.12) is not given in any text reviewed by the author. So its derivation will be presented here.

In order to obtain (A.12), first we need to write the Cauchy-Schwarz inequality (A.11) as an equality with a correction term \(\kappa\),
\[
\kappa + \left( \frac{1}{b-a} \int_a^b X(t)^\top Y(t)dt \right)^2 = \frac{1}{(b-a)^2} \left( \int_a^b X(t)^\top X(t)dt \right) \left( \int_a^b Y(t)^\top Y(t)dt \right). \tag{A.15}
\]
Now we will proceed by performing the dot products in (A.15),

\[
\kappa + \left( \frac{1}{b-a} \int_a^b (x_1(t)y_1(t) + x_2(t)y_2(t) + x_3(t)y_3(t) + \cdots + x_n(t)y_n(t))dt \right)^2 = \frac{1}{(b-a)^2} \left( \int_a^b (x_1(t)^2 + x_2(t)^2 + x_3(t)^2 + \cdots + x_n(t)^2)dt \right)^2 \left( \int_a^b (y_1(t)^2 + y_2(t)^2 + y_3(t)^2 + \cdots + y_n(t)^2)dt \right).
\]

(A.16)

Expanding (A.16) and expressing it using the summation notation,

\[
\kappa = \frac{1}{(b-a)^2} \sum_{r=1}^n \sum_{s=1}^n \left( \int_a^b x_r(t)^2 dt \int_a^b y_s(t)^2 dt \right)
- \frac{1}{(b-a)^2} \sum_{r=1}^n \sum_{s=1}^n \left( \int_a^b x_r(t)y_r(t)dt \right) \left( \int_a^b x_s(t)y_s(t)dt \right).
\]

(A.17)

Following [38], we need to pursue the symmetry, thus,

\[
\kappa = \frac{1}{2(b-a)^2} \sum_{r=1}^n \sum_{s=1}^n \left( \int_a^b x_r(t)^2 dt \int_a^b y_s(t)^2 dt + \int_a^b x_s(t)^2 dt \int_a^b y_r(t)^2 dt \right)
- \frac{1}{(b-a)^2} \sum_{r=1}^n \sum_{s=1}^n \left( \int_a^b x_r(t)y_r(t)dt \right) \left( \int_a^b x_s(t)y_s(t)dt \right).
\]

(A.18)

Since the integration variable is a dummy variable we can change it for any other character we want. Therefore, (A.18) becomes

\[
\kappa = \frac{1}{2(b-a)^2} \sum_{r=1}^n \sum_{s=1}^n \left( \int_a^b x_r(t)^2 dt \int_a^b y_s(\tau)^2 d\tau + \int_a^b x_s(\tau)^2 d\tau \int_a^b y_r(t)^2 dt \right)
- \frac{1}{(b-a)^2} \sum_{r=1}^n \sum_{s=1}^n \left( \int_a^b x_r(\tau)y_r(t)dt \right) \left( \int_a^b x_s(\tau)y_s(\tau)dt \right).
\]

(A.19)

Rearranging (A.19),

\[
\kappa = \frac{1}{2(b-a)^2} \sum_{r=1}^n \sum_{s=1}^n \left( \int_a^b \int_a^b (x_r(t)^2 y_s(\tau)^2 + x_s(\tau)^2 y_r(t)^2) d\tau dt \right)
- \frac{1}{(b-a)^2} \sum_{r=1}^n \sum_{s=1}^n \left( \int_a^b \int_a^b x_r(t)y_s(\tau)x_s(\tau)y_r(t) d\tau dt \right).
\]

(A.20)

Interchanging the summations and the integrals in (A.20) and rearranging it,

\[
\kappa = \frac{1}{2(b-a)^2} \int_a^b \int_a^b \sum_{r=1}^n \sum_{s=1}^n (x_r(t)^2 y_s(\tau)^2 - 2x_r(t)y_s(\tau)x_s(\tau)y_r(t) + x_s(\tau)^2 y_r(t)^2) d\tau dt.
\]

(A.21)
Finally factorizing (A.21), we obtain (A.12)

\[ \kappa = \frac{1}{2(b-a)^2} \int_a^b \int_a^b \sum_{r=1}^{n} \sum_{s=1}^{n} (y_r(t)x_s(\tau) - y_s(\tau)x_r(t))^2 dt d\tau. \]

### A.6 Complex \( L_2[a, b] \) space

The complex \( L_2[a, b] \) space is the set of all square-integrable complex functions in the interval \([a, b]\). This space will be used to obtain some power theories that are expressed in the complex plane. The condition for a complex function \( x(t) \) to be in the complex \( L_2[a, b] \) space is [22, 36],

\[ \int_a^b x(t) \overline{x(t)} dt < \infty. \] (A.22)

In this space, the inner product is defined as [22, 36],

\[ \langle \xi(t), \eta(t) \rangle \triangleq \frac{1}{b-a} \int_a^b \xi(t) \overline{\eta(t)} dt. \] (A.23)

From (A.23) the induced norm of \( L_2[a, b] \) is defined and it is given as,

\[ \| x(t) \| \triangleq \sqrt{\frac{1}{b-a} \int_a^b x(t) \overline{x(t)} dt}. \] (A.24)

Now, with the definition of the inner product (A.23) and the norm (A.24), we can define the Cauchy-Schwarz inequality for the complex \( L_2[a, b] \) space,

\[ \left| \frac{1}{b-a} \int_a^b \xi(t) \overline{\eta(t)} dt \right|^2 \leq \frac{1}{(b-a)^2} \left( \int_a^b |\xi(t)|^2 dt \right) \left( \int_a^b |\eta(t)|^2 dt \right). \] (A.25)

In order to have an equality in (A.25) a term must be added to the left hand side of it. And this has the following form,

\[ \frac{1}{2(b-a)^2} \int_a^b \int_a^b |\xi(t)\eta(\tau) - \xi(\tau)\eta(t)|^2 dt d\tau. \] (A.26)

With (A.26) we get the equality [39],

\[ \left| \frac{1}{b-a} \int_a^b \xi(t) \overline{\eta(t)} dt \right|^2 + \frac{1}{2(b-a)^2} \int_a^b \int_a^b |\xi(t)\eta(\tau) - \xi(\tau)\eta(t)|^2 dt d\tau = \frac{1}{(b-a)^2} \left( \int_a^b |\xi(t)|^2 dt \right) \left( \int_a^b |\eta(t)|^2 dt \right). \] (A.27)

The term (A.26) in (A.27) becomes zero when \( \xi(t) \) and \( \eta(t) \) are proportional or if one of them is zero.

The basis for the complex \( L_2[a, b] \) space are the complex exponentials. Thus, every function belonging to \( L_2[a, b] \) can be written as a linear combination of that basis,
Here, \( X_k \) are the complex numbers, \( f \) is the fundamental frequency and \( k \) is the harmonic number.

### A.7 Complex \( \ell_2 \) space

The complex \( \ell_2 \) space is the compound of all the complex square-summable sequences. Thus, a complex sequence \( \{X_k\}_{k=0}^{\infty} \) belongs to the complex \( \ell_2 \) space if

\[
\sum_{k=0}^{\infty} |X_k|^2 < \infty.
\]  

(A.29)

In the complex \( \ell_2 \) space, the inner product is defined as [38],

\[
\langle \{X_k\}_{k=0}^{\infty}, \{Y_k\}_{k=0}^{\infty} \rangle \triangleq \sum_{k=0}^{\infty} X_k Y_k.
\]  

(A.30)

With this definition of the inner product (A.30), we define the induced norm,

\[
\| \{X_k\}_{k=0}^{\infty} \| \triangleq \sqrt{\sum_{k=0}^{\infty} |X_k|^2}.
\]  

(A.31)

Now that the definitions of the inner product (A.30) and norm (A.31) are given, we define the Cauchy-Schwarz inequality for this Hilbert space,

\[
|\sum_{k=0}^{\infty} X_k Y_k|^2 \leq \sum_{k=0}^{\infty} |X_k|^2 \sum_{k=0}^{\infty} |Y_k|^2.
\]  

(A.32)

The Cauchy-Schwarz inequality (A.32) will become an equality if the following term is added to its left hand side, when the sequence has more than one element,

\[
\sum_{0 \leq p < k < \infty} |X_p Y_k - X_k Y_p|^2.
\]  

(A.33)

The equality that holds when we have more than one element in the sequence (A.29). The equality (A.34) is called the Lagrange’s identity for complex numbers [38],

\[
|\sum_{k=0}^{\infty} X_k Y_k|^2 + \sum_{0 \leq p < k < \infty} |X_p Y_k - X_k Y_p|^2 = \sum_{k=0}^{\infty} |X_k|^2 \sum_{k=0}^{\infty} |Y_k|^2.
\]  

(A.34)

For the case when we have just one element in the sequence (A.29), the equality is called the Fibonnaci-Brahmagupta equation (A.35) and it is given as follows [38],
\[ X_k = x_{Rk} + jx_{Ik}, \]
\[ Y_k = y_{Rk} + jy_{Ik}, \]
\[ |x_{Rk} + jx_{Ik}|^2 |y_{Rk} + jy_{Ik}|^2 = |x_{Rk}y_{Rk} + x_{Ik}y_{Ik}|^2 + |x_{Ik}y_{Rk} - x_{Rk}y_{Ik}|^2. \]  
(A.35)

**Remark:** The complex \( L_2 \) and \( \ell_2 \) represent the same kind of functions, since there is a one-to-one correspondence between the chosen basis of the complex \( L_2[a,b] \) space (A.28) and the complex sequences in \( \ell_2 \). Thus, using the Parseval’s theorem we can prove that the Lagrange’s identity (A.34) and the Fibonacci-Brahmagupta equation (A.35) are obtained from (A.27) [36]. This is done by expressing the complex functions \( x(t) \) and \( y(t) \) as a linear combination of complex exponentials (A.28). Then we substitute this linear combination in (A.27), simplifying and depending on the number of harmonics in the complex function we will obtain the Lagrange’s identity (A.34) or the Fibonacci-Brahmagupta equation (A.35) [36].
Appendix B

Industrial SVC power circuit

Figure B.1: Static VAr Compensator testbench developed at Diram.
Figure B.2: SVC protections.

Figure B.3: Control circuits and line reactor.

Figure B.4: Thyristor controlled reactor.
Figure B.5: Second harmonic filter.

Figure B.6: Third, fourth and fifth harmonic filters.
# Appendix C

## Control circuit components

<table>
<thead>
<tr>
<th>Component</th>
<th>Model</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hall effect current sensor</td>
<td>LA 55-P</td>
<td><img src="image" alt="Hall effect current sensor image" /></td>
</tr>
<tr>
<td>Transformer</td>
<td>14A-30-20</td>
<td><img src="image" alt="Transformer image" /></td>
</tr>
<tr>
<td>Rectifiers</td>
<td>TRS404L</td>
<td><img src="image" alt="Rectifiers image" /></td>
</tr>
<tr>
<td>Filter capacitor</td>
<td>2200 $uF$</td>
<td><img src="image" alt="Filter capacitor image" /></td>
</tr>
</tbody>
</table>

Table C.1: Sensor board component
## Linear regulators
- LM7805
- LM7812
- LM7912

### Table C.2: Liner power supply board components

<table>
<thead>
<tr>
<th>Component</th>
<th>Model</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isolation Transformer</td>
<td>44159</td>
<td><img src="iso_transformer.png" alt="Image" /></td>
</tr>
<tr>
<td>Varistor</td>
<td>V25S150P</td>
<td><img src="varistor.png" alt="Image" /></td>
</tr>
<tr>
<td>Operational amplifier</td>
<td>OPA2277A</td>
<td><img src="opamp.png" alt="Image" /></td>
</tr>
</tbody>
</table>
| Trimmer Potentiometers| • 100 kΩ  
                       | • 10 kΩ  
                       | • 100 Ω  | ![Image](potentiometer.png) |
| Resistor 1/4 W       | • 50 kΩ  
                       | • 100 kΩ | ![Image](resistor.png) |
| Voltage reference    | LM285LPR-2-5 | ![Image](vref.png) |
Table C.3: DSP and signal conditioning components

<table>
<thead>
<tr>
<th>Component</th>
<th>Model</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zener diode</td>
<td>1N4728</td>
<td></td>
</tr>
<tr>
<td>DSP experimental kit</td>
<td>TMDSDOCK28335</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Component</th>
<th>Model</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>AND gate</td>
<td>74HCT08</td>
<td><img src="image" alt="AND gate" /></td>
</tr>
<tr>
<td>Optocoupler</td>
<td>MOC3021M</td>
<td><img src="image" alt="Optocoupler" /></td>
</tr>
<tr>
<td>Resistor 1/4 W</td>
<td>330 Ω</td>
<td><img src="image" alt="Resistor" /></td>
</tr>
<tr>
<td>Resistor 5 W</td>
<td>300 Ω</td>
<td><img src="image" alt="Resistor" /></td>
</tr>
<tr>
<td>Relay driver</td>
<td>ULN2003A</td>
<td><img src="image" alt="Relay driver" /></td>
</tr>
<tr>
<td>Relay</td>
<td>JS1A-12V-F</td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td>-----------</td>
<td></td>
</tr>
</tbody>
</table>

Table C.4: Power interface components
Appendix D

Control circuit layouts

Figure D.1: Sensor board layout.

Figure D.2: Linear power supply layout.
Figure D.3: DSP and signal conditioning bottom layout.

Figure D.4: DSP and signal conditioning top layout.
Figure D.5: Power interface bottom layout.

Figure D.6: Power interface top layout.
Appendix E

Implementation code

The TI-RTOS kernel can be configured using a graphical user interface to set the threads. The system overview is depicted in Fig. E.1.

![Figure E.1: TI-RTOS kernel setup.](image)

In the next sections we will describe how were configured the four threads used in this thesis. And we will also add the functions that each of this threads call once the RTOS schedule them.

E.1 Clock

With the clock thread we create a software interrupt that mimics a timer. The implemented clock ticks will be every 83 $\mu$s. The setup of the clock ticks is depicted in Fig. E.2.
Now with these clockticks we implement a software driven timer. This clock thread will be called every clock tick. Thus, it will be calling the startAdcSeq function at 12.048 kHz. How this was set up is depicted in Fig. E.3.

The function called by the clock is in the Listing E.1.

Listing E.1: Function startAdcSeq

```c
void startAdcSeq(UArg a0, UArg a1)
{
    //ADC
    AdcRegs.ADCTRL2.bit.RST_SEQ1 = 1;  // Reset SEQ1
    AdcRegs.ADCTRL2.bit.SOC_SEQ1 = 1;  // Initiate the ADC
}
```

E.2 ADC HWI

The ADC HWI waits until the measurements are ready to be read. When these measurements are ready, the interrupt number 37 from the peripheral interrupt expansion vector table [40]. Using this interrupt number we set up the ADC HWI. This set up is depicted in Fig. E.4.
E.3. SWI

Figure E.4: ADC HWI set up.

The function called by the ADC HWI is in the Listing E.2.

Listing E.2: Function readAdc

```c
Uint16 voltage[3];
Uint16 current[3];
void readAdc(UArg arg0)
{
    voltage[0] = AdcRegs.ADCRESULT0 >> 4; //ADC Pin A0
    current[0] = AdcRegs.ADCRESULT6 >> 4; //ADC Pin A3
    current[1] = AdcRegs.ADCRESULT8 >> 4; //ADC Pin A4
    //Swi synchronization.
    Swi_post(Synchronization);
    AdcRegs.ADCST.bit.INT_SEQ1_CLR = 1; // Clear INT SEQ1 bit
}
```

E.3 SWI

At the end of the ADC HWI given in the Listing E.2 there is a posted SWI calling Synchronization. This SWI will transform the voltages and currents into the $\alpha - \beta$ and the $d - q$ frame respectively. With that information it will compute the grid angle, the thyristors firing angle and it will fire the thyristors. The set up of this SWI is depicted in Fig. E.5.

Figure E.5: SWI set up.
The function called by the SWI is in the Listing E.3.

Listing E.3: Function transSynchPuls

```c
float32 firingAnglep; // Stores positive angles
float32 firingAnglem; // Stores negative angles
Uint16 flagOnPulses = 0; // Flag to set on the pulses
Uint16 flagOnClock = 0;
Uint16 flagOnPI = 0;
//--------------------------------------------------
// FSM using a linked list
//--------------------------------------------------
typedef struct State
{
    Uint16 Estado; // Thyristor number (Output)
    const struct State *Next; // Next state
} STyp;

#define T1 &fsmAB[0]
#define T2 &fsmCA[0]
#define T3 &fsmBC[0]
#define T4 &fsmAB[1]
#define T5 &fsmCA[1]
#define T6 &fsmBC[1]

// 3 FSM with 2 States
STyp fsmAB[2] =
{
    {1,T4}, // State Thyristor 1
    {4,T1}  // State Thyristor 4
};

STyp fsmBC[2] =
{
    {3,T6}, // State Thyristor 3
    {6,T3}  // State Thyristor 6
};

STyp fsmCA[2] =
{
    {2,T5}, // State Thyristor 2
    {5,T2}  // State Thyristor 5
};

// Initial states definition
STyp* ptAB = T1;
STyp* ptCA = T2;
STyp* ptBC = T3;

void transSynchPuls(void)
{
    // Transformations:
```
```
E.3. SWI

--------------------------------
float32 v_a, v_b, v_c;    //Voltages in the abc frame
float32 i_a, i_b, i_c;    //Currents in the abc frame
float32 _alpha, _beta;   //Currents in the alpha-beta frame
float32 _d, _q;          //Currents in the d-q frame.
float32 theta;           //Angle in the alpha-beta plane
float32 thetaAB , thetaBC, thetaCA;   //Line to line angles
float32 piAngle;        //Variable to save the PI controller angle

//Removing the offset of the voltages and currents
//The offset is 1.66V. Thus, (1.66/3)*(4095)=2265.9
//
v_a = (float32)voltage[0] - 2265.9;
v_b = (float32)voltage[1] - 2265.9;
v_c = (float32)voltage[2] - 2265.9;

//Obtaining the angle in the alpha-beta plane
//Computing the voltages in the alpha-beta frame
clarkeTransformation(v_a, v_b, v_c, &v_alpha, &v_beta);
//Computing the angle
theta = atan2(v_beta, v_alpha);

//Obtaining the current in the d-q plane
//computing the currents in the alpha-beta frame
clarkeTransformation(i_a, i_b, i_c, &i_alpha, &i_beta);
//Calculating the currents in the d-q frame
parkTransformation(theta, i_alpha, i_beta, &i_d, &i_q);

---------------------------------
// Synchronization:
//-------------------------------
synchLineToLine(theta, &thetaAB, &thetaBC, &thetaCA);

if(flagOnPulses != 0)
{
    //-------------------------------
    // PI Controller:
    //-------------------------------
    piAngle = pi_Controller(i_q);  //PI controller @12kHz

    if(flagOnPI == 1)
    {
        firingAnglep = piAngle;
        firingAnglem = firingAnglep - 3.14159265;
    }
}
APPENDIX E. IMPLEMENTATION CODE

105  // Set pulses on AB
106  // Set pulses on AB
107  if ( ( thetaAB > 0 ) && ( thetaAB < 1.570796327 ) )
108  {
109    if((thetaAB >= firingAnglep) && (ptAB->Estado == 1))
110      {
111        GpioDataRegs.GPCSET.bit.GPIO84 = 1; //Thy 1 on
112        Timer_start(pulseSeqTimer); //Start the timer
113        flagOnPI = 0;
114      }
115  }
116
117  if((thetaAB > -3.1415926536) && (thetaAB < -1.570796327))
118  {
119    if((thetaAB > firingAnglem) && (ptAB->Estado == 4))
120      {
121        GpioDataRegs.GPASET.bit.GPIO15 = 1; //Thy 4
122        Timer_start(pulseSeqTimer); //Start the timer
123      }
124  }
125
126  // Set pulses on BC
127  // Set pulses on BC
128  if((thetaBC > 0) && (thetaBC < 1.570796327))
129  {
130    if((thetaBC >= firingAnglep) && (ptBC->Estado == 3))
131      {
132        GpioDataRegs.GPASET.bit.GPIO12 = 1; //Thy 3
133        Timer_start(pulseSeqTimer); //Start the timer
134      }
135  }
136
137  if((thetaBC > -3.1415926536) && (thetaBC < -1.570796327))
138  {
139    if((thetaBC >= firingAnglem) && (ptBC->Estado == 6))
140      {
141        GpioDataRegs.GPASET.bit.GPIO26 = 1; //Thy 6
142        Timer_start(pulseSeqTimer); //Start the timer
143        flagOnPI = 1;
144      }
145  }
146
147  // Set pulses on CA
148  // Set pulses on CA
149  //------ Set pulses on BC ------
150  //------ Set pulses on BC ------
if((thetaCA > 0) && (thetaCA < 1.570796327))
{
    //GpioDataRegs.GPATOGGLE.bit.GPIO24 = 1;//Debug
    if((thetaCA >= firingAnglep) && (ptCA->Estado == 5))
    {
        GpioDataRegs.GPASET.bit.GPIO24 = 1; //Thy 5
        ptCA = ptCA->Next;
        Timer_start(pulseSeqTimer); //Start the timer
    }
}

if((thetaCA > -3.1415926536) && (thetaCA < -1.570796327))
{
    //GpioDataRegs.GPCTOGGLE.bit.GPIO86 = 1;
    if((thetaCA >= firingAnglem) && (ptCA->Estado == 2))
    {
        GpioDataRegs.GPASET.bit.GPIO86 = 1; //Thy 2
        ptCA = ptCA->Next;
        Timer_start(pulseSeqTimer); //Start the timer
    }
}

if(flagOnClock == 0)
{
    Clock_start(pulseTCRTimer);
    flagOnClock = 1;
}

//-----------------------------------------------------
// clarkeTransformation:
//-----------------------------------------------------
// Computes the power invariant Clarke transformation:
// inputs: Voltages or Currents in the abc domain,
// pointers to store the results
void clarkeTransformation(float32 xa, float32 xb, float32 xc,
float32* xalpha, float32* xbeta)
{
    //Storing the alpha value
    *xalpha = (xa - 0.5*xb - 0.5*xc)*(0.8165);
    //Storing the beta value
    *xbeta = (0.866*xb - 0.866*xc)*(0.8165);
}

//-----------------------------------------------------
// parkTransformation:
//-----------------------------------------------------
//Computes the power invariant Park transformation:
// inputs: Voltages or Currents in the alpha-beta domain, pointers to
// store the results
void parkTransformation(float32 radians, float32 xalpha,
float32 xbeta, float32* xd, float32* xg)
float32 abCos, abSin;
sincos(radians, &abSin, &abCos);
    //Storing the d value
    *xd = abCos * xalpha + abSin * xbeta;
    //Storing the q value
    *xq = -abSin * xalpha + abCos * xbeta;
}

void synchLineToLine(float32 angle, float32* angleAB,
    float32 angleBC, float32* angleCA)
{
    *angleAB = angle + 0.5235987; //angle + Pi/6
    *angleCA = angle + 2.61799389; //angle + 5Pi/6
    *angleBC = angle - 1.5707963268; //angle - Pi/2

    if( *angleAB > 3.1415926 )
    {
        *angleAB = *angleAB - 6.2831853;
    }

    if( *angleCA > 3.1415926 )
    {
        *angleCA = *angleCA - 6.2831853;
    }

    if( *angleBC < -3.1415926 )
    {
        *angleBC = *angleBC + 6.2831853;
    }

    float32 pi_Controller(float32 i_q)
    {
        float32 Angle, stPI;
        float32 k = 0.0003;
        float32 T = 0.01;
        float32 f = 12000;
        //PI controller using backward Euler
        static float32 out_A = 0.0;
        stPI = out_A + (k/(T*f)) * i_q;
        stPI = (stPI < 0.023596) ? 0.023596 : ((stPI > 2.117992) ?
         2.117992 : stPI);
        out_A = stPI;
        stPI += k * i_q;
        stPI = (stPI < 0.023596) ? 0.023596 : ((stPI > 2.117992) ?
         2.117992 : stPI);

        //Look up table
        const float32 lkupTbl[10][2] = {{0,1.570795},
            {0.007043196,1.396262222},
            {0.05534192,1.221729444},
            ...}
E.4 Timer HWI

The Timer HWI will be posted by the SWI listed in E.3, every time a thyristor is fired. This Timer will assure that the thyristor on pulse will be on for 30 $\mu$s. This time was selected because the thyristors need a 3 $\mu$s pulse to turn on. The Timer HWI set up is depicted in Fig. E.6.

```c
if (stPI <= lkupTbl[0][0])
{
    Angle = lkupTbl[0][1];
} else if (stPI >= lkupTbl[10 - 1][0])
{
    Angle = lkupTbl[10 - 1][1];
} else
{
    int i;
    for (i = 1; i < 10; i++) {
        if (stPI < lkupTbl[i][0])
        {
            Angle = lkupTbl[i-1][1] + (lkupTbl[i][1] - lkupTbl[i -1][1]) * (stPI - lkupTbl[i-1][0]) / (lkupTbl[i ][0] - lkupTbl[i-1][0]);
            break;
        }
    }
    return Angle;
}
```

Figure E.6: Timer HWI set up.
The function called by the Timer HWI is in the Listing E.4.

Listing E.4: Function pulseOffSeq

```
void pulseOffSeq(void)
{
    //Turning off AB+
    if( ptAB->Estado == 4 )
    {
        GpioDataRegs.GPCCLEAR.bit.GPIO84 = 1; //Thy 1 off
    }

    //Turning off AB-
    if( ptAB->Estado == 1 )
    {
        GpioDataRegs.GPACLEAR.bit.GPIO15 = 1; //Thy 4 off
    }

    //Turning off BC+
    if( ptBC->Estado == 6 )
    {
        GpioDataRegs.GPACLEAR.bit.GPIO12 = 1; //Thy 3 off
    }

    //Turning off BC-
    if( ptBC->Estado == 3 )
    {
        GpioDataRegs.GPACLEAR.bit.GPIO26 = 1; //Thy 6 off
    }

    //Turning off CA-
    if( ptCA->Estado == 5 )
    {
        GpioDataRegs.GPCCCLEAR.bit.GPIO86 = 1; //Thy 2 off
    }

    //Turning off CA+
    if( ptCA->Estado == 2 )
    {
        GpioDataRegs.GPACLEAR.bit.GPIO24 = 1; //Thy 5 off
    }
}
```
Appendix F

Simulation results

The values used to simulate the industrial SVC are the same as the ones given in Table 3.1.

F.1 Open loop simulation results

The whole plots depicted in this section will be the simulation of the figures depicted in Section 5.1.

Figure F.1: Simulation of the four FHF.

[Diagram of voltage and current waveforms]
Figure F.2: Simulation of the TCR with an $\alpha = 30^\circ$.

Figure F.3: Simulation of the TCR with an $\alpha = 45^\circ$. 
F.1. Open Loop Simulation Results

Figure F.4: Simulation of the TCR with an $\alpha = 60^\circ$.

Figure F.5: Simulation of the TCR with an $\alpha = 75^\circ$. 
F.2 Closed loop simulation results

In the close loop simulation results we will present the behavior of the industrial SVC just when the capacitor bank is being connected and disconnected.

Using the load powers from Table 5.6, the three phase power definition for sinusoidal and balanced loads from equations (2.86)-(2.87) and assuming that the loaded induction motor in steady state and the capacitor bank are wye connected RL and RC circuits respectively. The computed values to simulate its steady state behavior are given in Table F.1.

<table>
<thead>
<tr>
<th>Load</th>
<th>Resistance [Ω]</th>
<th>Inductance [mH]</th>
<th>Capacitance [mF]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Induction motor</td>
<td>6.7422</td>
<td>27.786</td>
<td>-</td>
</tr>
<tr>
<td>Capacitor bank</td>
<td>0.0718</td>
<td>-</td>
<td>1.8227</td>
</tr>
</tbody>
</table>

Table F.1: Simulation parameters of the loads in steady state.

Figure F.6: Simulation of the current at the PCC when the capacitor bank is turned on.
Figure F.7: Simulation of the TCR current when the capacitor bank is turned on.

Figure F.8: Simulation of the current at the PCC when the capacitor bank is turned off.
Figure F.9: Simulation of the TCR current when the capacitor bank is turned off.
Bibliography


Curriculum Vitae

Roberto García Rochín was born in Mazatlán, México, on May 14th, 1992. He earned the Mechatronics Engineering degree with honors from the Instituto Tecnológico de Culiacán in December 2015. Where he worked on the Modeling and Control of Ultrasonic Motors in collaboration with the University of West England and the Technical University of Munich. He was accepted in the Industrial Consortium to Foster Applied Research in Mexico in July 2016.