CHARACTERIZATION AND VALIDATION OF A HYSTERETIC DYNAMIC NON-LINEAR PIEZOCERAMIC ACTUATOR MODEL

TESIS
PRESENTADA COMO REQUISITO PARCIAL PARA OBTENER EL GRADO ACADÉMICO DE
MAESTRO EN CIENCIAS ESPECIALIDAD EN SISTEMAS DE MANUFACTURA

POR:
MARIO JOSÉ QUANT JO

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Mayo 2009
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INSTITUTO TECNOLÓGICO Y DE ESTUDIOS SUPERIORES DE MONTERREY

Mayo 2009
Abstract

The use of smart materials as actuators and sensors has experienced a great expansion in recent years, mainly in the aerospace, automotive, civil engineering and medical fields. From all of the existent smart materials, piezoelectric ceramics have gained significant attention among researchers, mainly due to their fast response operation and considerable strain and force output. Their use as actuators can be divided into three main categories: positioners, motors and vibration suppressors. Limitations on the use of piezoelectric materials include various nonlinearities in their operational behaviour, such as hysteresis, material nonlinearities, frequency response, creep, aging and thermal behaviour.

This thesis presents an improved model for piezoceramic actuators, which accounts for hysteresis, dynamic response and nonlinearities. The hysteresis model is based on the widely used General Maxwell Slip model. An electro-mechanical non-linear model replaces the linear constitutive equations commonly used, and a linear second order model compensates the frequency response of the actuator.

A specific piezoceramic actuator is selected for full and detailed experimental characterization. The model is built in a Matlab/Simulink environment, and validated via experimental results. Based on the same formulation, two other models are also proposed: one that is intended to operate within a force-controlled scheme (as opposite to the first model, which is based in a displacement/position control), and a piezoceramic actuator inverse model, implemented for an open-loop control scheme, which compensates nonlinearities to obtain a “linearized” behaviour of the actuator. Simulations are carried out using open and closed loop control theory, including a mechanical interaction with a finite element model of a cantilever beam.
I would like to dedicate this thesis to my family, my mother and father, and my two brothers, without whom I would have never accomplished this.
Acknowledgements

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Glossary

Roman letters

\( b \) Damping \([\text{Ns/m}^2]\)
\( b_p \) Width of piezoceramic actuator \([\text{m}]\)
\( C \) Linear capacitance \([\text{C/m}]\)
\( C_i \) Capacitance value of element \([\text{F}]\)
\( c \) Damping of beam \([\text{N-s/m}]\)
\( c_{ij} \) Matrix of elastic stiffness coefficients \([\text{N/m}^2]\)
\( D \) Vector of electric displacement \([\text{C/m}^2]\)
\( d_{mi} \) Matrix of piezoelectric strain (or charge) coefficients \([\text{C/N}]\) or \([\text{m/V}]\)
\( E \) Vector of applied field \([\text{V/m}]\)
\( e_{mi} \) Inverse matrix of piezoelectric voltage coefficients \([\text{C/m}^2]\) or \([\text{N/Vm}]\)
\( F_a \) Output force of the PA \([\text{N}]\)
\( F_b \) Blocked force \([\text{N}]\)
\( F_{ext} \) External force \([\text{N}]\)
\( F_i \) Reaction force \([\text{N}]\)
\( F_T \) Transformer force \([\text{N}]\)
\( F_t \) Force applied at the tip of the beam \([\text{N}]\)
\( f \) Operating frequency \([\text{Hz}]\)
\( f_a \) Anti-resonant frequency \([\text{Hz}]\)
\( f_i \) Breakaway friction force of the block \([\text{N}]\)
\( f_r \) Resonant frequency \([\text{Hz}]\)
\( g_{mi} \) Matrix of piezoelectric voltage coefficients \([\text{m}^2/\text{C}]\) or \([\text{Vm/N}]\)
\( h_{mi} \) Inverse matrix of piezoelectric strain (or charge) coefficients \([\text{N/C}]\) or \([\text{V/m}]\)
\( K_{eq}^3 \) Relative dielectric constant []
\( k \) Stiffness \([\text{N/m}^2]\)
\( k_{eq} \) Equivalent stiffness of beam \([\text{N/m}]\)
\( k_i \) Linear stiffness of the spring \([\text{N/m}]\)
\( l_a \) Actuator length \([\text{m}]\)
\( l_b \) Beam length \([\text{m}]\)
\( l_p \) Length of piezoceramic actuator \([\text{m}]\)
\( M \) Generated moment due to actuator force [Nm]
\( m \) Mass [kg]
\( m_{eq} \) Equivalent mass of beam [kg]
\( N_i \) Normal force acting on the block [N]
\( n_{em} \) Electro-mechanical couple [C/m] or [N/V]
\( P \) Power [W]
\( q \) Total charge [C]
\( \dot{q} \) Total current [C/s]
\( q_{bi} \) Charge level for an element [C]
\( \dot{q}_c \) Capacitor current [C/s]
\( \dot{q}_T \) Transformer current [C/s]
\( S \) Strain vector [m/m]
\( s_{ij} \) Matrix of elastic compliance coefficients [m^2/N]
\( s_j \) Slope [N/m] or [V/C]
\( T \) Stress vector [N/m^2]
\( t_a \) Actuator thickness [m]
\( t_b \) Beam thickness [m]
\( t_p \) Thickness of piezoceramic actuator [m]
\( V \) Voltage [V]
\( V_H \) Hysteresis voltage [V]
\( V_{i} \) Output voltage for an element [V]
\( V_{in} \) Input voltage [V]
\( V_T \) Transformer voltage [V]
\( v_i \) Breakaway voltage [V]
\( x \) Displacement [m]
\( \dot{x} \) Velocity [m/s]
\( \ddot{x} \) Acceleration [m/s^2]
\( x_a \) Starting position of the actuator from the fixed side of the beam [m]
\( x_{bi} \) Current position of the block [m]
\( Y_{11} \) Elastic modulus along axis-1 [N/m^2]
\( Z_t \) Tip displacement along z axis [m]

**Greek letters**
\( \beta_{mk} \) Impermittivity component [m/F]
\( \varepsilon_a \) Actuator’s average strain along the x axis [m/m]
\( \varepsilon_{mk} \) Permittivity component [F/m]
\( \varepsilon_0 \) Permittivity of free space [C/Vm]
\( \Lambda \) Free strain [m/m]
\( \mu_i \) Friction coefficient []
Subscripts

- $i, j$: a) Indexes from 1 to 6, b) $i_{th}$ elasto-slide element, c) $j_{th}$ segment
- $m, k$: Indexes from 1 to 3
- $n$: Number of elasto-slide elements

Superscripts

- $D$: Zero or constant electric displacement (open circuit)
- $E$: Zero or constant electric field (short circuit)
- $S$: Zero or constant strain
- $T$: Zero or constant stress

Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANSI</td>
<td>American National Standards Institute</td>
</tr>
<tr>
<td>BaSTO</td>
<td>Barium Strontium Titanate</td>
</tr>
<tr>
<td>CPM</td>
<td>Classical Preisach Model</td>
</tr>
<tr>
<td>DNLRLX</td>
<td>Dynamic Non-Linear Regression with direct application of eXcitation</td>
</tr>
<tr>
<td>GMS</td>
<td>Generalized Maxwell Slip model</td>
</tr>
<tr>
<td>ER</td>
<td>Electro Rheological</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Model</td>
</tr>
<tr>
<td>FRF</td>
<td>Frequency Response Function</td>
</tr>
<tr>
<td>FSR</td>
<td>Full Scale Range</td>
</tr>
<tr>
<td>HUMS</td>
<td>Health and Usage Monitoring Systems</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>IRE</td>
<td>Institute of Radio Engineers</td>
</tr>
<tr>
<td>MR</td>
<td>Magneto Rheological</td>
</tr>
<tr>
<td>MRC</td>
<td>Maxwell Resistive Capacitive model</td>
</tr>
<tr>
<td>NARMAX</td>
<td>Non-linear Auto-Regressive Moving Average with eXogenous inputs</td>
</tr>
<tr>
<td>NiTiNol</td>
<td>Nickel Titanium alloy</td>
</tr>
<tr>
<td>NMAX</td>
<td>Non-linear Moving Average model with eXogenous inputs</td>
</tr>
<tr>
<td>PA</td>
<td>Piezoceramic Actuator</td>
</tr>
<tr>
<td>PD</td>
<td>Proportional Derivative</td>
</tr>
<tr>
<td>P-I</td>
<td>Prandtl-Ishlinskii model</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional Integral Derivative</td>
</tr>
<tr>
<td>PMN</td>
<td>Lead Magnesium Niobate</td>
</tr>
<tr>
<td>PVDF</td>
<td>Polyvinylidene Fluoride</td>
</tr>
<tr>
<td>PZT</td>
<td>Lead Zirconate Titanate</td>
</tr>
<tr>
<td>SMA</td>
<td>Shape Memory Alloy</td>
</tr>
<tr>
<td>TerFeNol-D</td>
<td>Terbium Iron</td>
</tr>
<tr>
<td>TF</td>
<td>Transfer Function</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

The use of smart materials as actuators and sensors has experienced a great expansion in recent years, mainly in the aerospace, automotive, civil engineering and medical fields [1-12]. Applications are only limited to the imagination of researchers and engineers, thus they promise to change the functionality and design of many products. Due to great versatility, such materials have unique inherent properties which are fit for specialized application. For example, there are different kinds of smart materials suitable for interacting with mechanical structures, such as: piezoelectric, shape memory alloys, electro/magneto-strictive, electro/magneto-rheological fluids, etc. [4, 13-15], thereby, a careful selection must be performed for the application in mind.

From all of the existent smart materials, piezoelectric ceramics have gained significant attention among researchers. Their use as actuators can be divided into three main categories: positioners, motors and vibration suppressors. Applications range from structural noise and vibration control in commercial, industrial, military, and scientific equipment to medical diagnostic imaging, non-destructive testing, health monitoring of machinery, MEMS technology, and precision manufacturing [18]. Specific applications for piezoceramic actuators include: loudspeakers, piezoelectric motors, laser mirror alignment systems, inkjet printers, diesel engines fuel injectors, atomic force microscopes, active control vibration and XY stages for micro-scanning.

A key factor in the use of piezoelectric materials is the precision at which they operate. A standard on piezoelectricity was published in 1979 by the IEEE/ANSI [19], which states the basic linear constitutive equations that rule their behaviour. However, it is well known that piezoelectric materials present a non-linear behaviour, mainly due to hysteresis between the input voltage and generated electric charge. For this reason, in the past years, several models have been proposed for piezoelectric materials which account mainly for the hysteresis. These hysteresis models include the Classical Preisach Model (CPM) and variations, the Generalized Maxwell Slip (GMS) model also known as the Maxwell Resistive Capacitive (MRC) model, the Prandtl-Ishlinskii (P-I), Bouc-Wen, Duhem, LuGre and Leuven models among others.
Additional piezoelectric behaviour which is often neglected include: frequency response, non-linear input dependence, creep, aging and thermal behaviour. Therefore, the development of a model that could account for the aforementioned properties, including the proper electro-mechanical relation that define piezoelectric materials, would allow an enhanced use of the materials in specific applications where accuracy is of high importance.

In this document, a specific piezoceramic actuator is selected for full characterization of a new model that accounts for hysteresis behaviour, dynamic response and nonlinearities. The developed model is then validated by comparison with experimental results. An alternate model that focuses on force output instead of strain is also formulated; as well as an inverse model of the piezoceramic. This inverse model linearizes the piezoceramic behaviour, compensating properties such as hysteresis, which can be of great significance when working in an open-loop control scheme.

1.1. Motivation

Nowadays, the use of piezoceramic actuators has been under intense research given their great versatility. Because of their high operating frequency range (some can reach the GHz limit), they are the most widely employed form of smart material actuator [14]. Another advantage is that they can be manufactured in several forms (e.g. patches, disks, tubes) and sizes, and they can be “trained” to work in different configurations. Also, new applications are proposed constantly.

However, structural models often focus on a specific type of behaviour (i.e. hysteresis, dynamicity, creep, thermal, non-linear coefficients, etc.), neglecting other significant effects. For these reasons, this investigation will focus on the development of a comprehensive model that accounts for the most significant factors influencing structural behaviour: hysteresis, dynamics, and non-linear coefficients.
1.2. Objectives

- To obtain a full static and dynamic characterization of a piezoceramic actuator that could also be used for other types of piezoelectric materials.

- To develop an explicit model of a piezoceramic actuator that accounts for hysteresis, electro-mechanical dynamics and nonlinearities of the material and piezoelectric effect.

- To perform an experimental validation of the developed model.

- To obtain an inverse model of the piezoceramic actuator suitable for use within a control loop, thus increasing the robustness of a simulation.

1.3. Contribution

In this document, a detailed characterization procedure for a piezoceramic actuator is presented, as well as its experimental validation. The obtained data is used to build a more realistic electro-mechanical dynamic model of the actuator, which also includes material and piezoelectric non-linearity, and hysteresis. This model represents a variation and improvement of other previously published [22-24]. In addition, the proposed model is intended to operate within a force-controlled matter, in addition to existing models based on displacement/position control.

The generated model will provide a general basis for realistic simulations of smart structures (i.e. predictive structural behaviour within a control loop system), as well as theoretical grounds for further investigations.

1.4. Overview

Chapter 2 presents a background on smart materials and smart structures. A summarized comparison between different types of smart materials is shown and an introduction to piezoelectricity is presented. A dedicated research on the applications and investigations on piezoceramic actuators is summarized, as well as research on piezoceramic actuators and hysteresis modelling.
In Chapter 3, first the constitutive equations that rule the piezoelectric materials are defined. Then it resumes the modelling of the piezoceramic actuator. Starting with the Generalized Maxwell Slip (GMS) model, which explains hysteresis; and then, with an electro-mechanical dynamic model which includes material nonlinearities.

Chapter 4 focuses first on the characterization techniques for piezoceramics, based on quasi-static and dynamic measurements. Afterwards, the equipment and experimental set-up required for measurement purposes is defined. Obtained experimental data from measurements and results are also presented.

Chapter 5 presents the characterization resulting values which are used to develop a Matlab/Simulink block model. This model is then validated comparing its response with previously-obtained experimental data. An alternate model that focuses on the output force instead of the output strain, and an inverse model for open-loop operation are presented.

In Chapter 6, a set of simulations in Matlab/Simulink are developed. First, an interaction between a piezoceramic actuator patch with a mechanical system (cantilever beam) is formulated. A finite element model (FEM) is used to simulate and complete the definition of the interaction. Afterwards, an open-loop control simulation for strain/position follower is explained, demonstrating the use of an inverse model to linearize the output behaviour of the actuator. Also closed-loop control simulations are presented, based on the whole mechanical system for vibration attenuation and force target follower control.

Final conclusions resulting from the characterization, modelling procedure, validation and simulations are presented in Chapter 7. Future work opportunities based on the present investigation are also stated.
2.1. Smart Materials

Technology and science have made great developments in design of machinery and electronics based on structural materials (i.e. aluminium, steel, copper), for which main sought property is strength. Nowadays, scientists have developed special materials which have unique properties that can be manipulated according to required specifications. These are called Smart Materials.

A Smart Material is a material that has one or more properties (mechanical, optical, electric, electromagnetic, etc.) that can be modified (shape, stiffness, viscosity, damping, etc.) via an external stimuli (voltage, temperature, stress, etc.) in a predictable, controlled and reversible manner. Depending on the relationship between properties and stimulus, we can consider a variety of effects as shown in Table 1. These types of materials are mainly transducers, meaning that they can exchange energy from one type to another (i.e. mechanical to electrical). According to desired effects, a smart material can be specifically developed. Varieties of these materials already exist, and are being researched extensively.

<table>
<thead>
<tr>
<th>INPUT</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Charge/Current</td>
</tr>
<tr>
<td>Electric Field</td>
<td>Permittivity-Conductivity</td>
</tr>
<tr>
<td>Magnetic Field</td>
<td>Magnetic-Electric effect</td>
</tr>
<tr>
<td>Stress</td>
<td>Direct piezo effect</td>
</tr>
<tr>
<td>Heat</td>
<td>Pyroelectric effect</td>
</tr>
<tr>
<td>Light</td>
<td>Photovoltaic effect</td>
</tr>
</tbody>
</table>

Table 1 - Transducer relationships [25]
Some of the principal existent smart materials that involve a mechanical behaviour are:

### 2.1.1. Piezoelectric Materials

These materials can undergo surface elongation (strain) when an electric field is applied across them (converse effect), also producing an electric charge under application of a stress (direct effect) [25]. Suitable designed structures made from these materials can therefore be tailor-made so to bend, expand or contract when a voltage is applied. Their applications include sensors and actuators due to the piezoelectric effect. Some of the advantages of piezoelectric materials are that they can achieve up to 0.2 % strain [15, 16], and can be stacked to obtain a greater output displacement or force. They have a low thermal coefficient and cover a wide frequency spectrum, even on the range of Giga-Hz. Some disadvantages are that they are very fragile during manipulation, need large voltages to operate, and present a considerable degree of hysteresis [17]. Many actuators and sensors are built with Lead Zirconate Titanate (PZT), the most common piezoelectric material. Piezoelectrics have a wide variety of applications, starting from daily-use objects such as lighters or guitar tuners, to engineer applications such as air-bag sensors, accelerometers and structural vibrators.

### 2.1.2. Shape Memory Alloys (SMA)

SMAs are thermo-responsive materials where deformation can be induced and recovered through (current-controlled) temperature changes. This deformation occurs because they suffer a phase transformation at certain temperature levels. These materials can reach a high level of force and displacement when stimulated and are mainly used as actuators in the form of wire, strips or films. Advantages are simplicity of use and bio-compatibility; and disadvantages are high hysteresis and low operating frequency, mainly due to cooling of the material. The most commercial SMA is Nitinol (Nickel Titanium alloy) which can deform up to 8% [15, 25]. Other common materials are CuZnAl and CuAlNi. Several companies sell these materials, such as: Dynalloy, SMA-Inc., TiNi Alloy Co., Jergens Inc., Mitsubishi Heavy Industries. They are being used in aeronautical applications such as in manipulation of flexible wing surfaces; in the medical area as surgical tools like bone plates or as robotic muscle wires.
2.1.3. Magneto-strictive

This kind of materials stretches when exposed to a magnetic field, exhibiting the Joule effect or magneto-striction. This occurs because magnetic domains in the material align with the magnetic field. In an opposite way, when a strain is induced in the material, its magnetic energy changes under the magneto-mechanical effect (Villari effect). Advantages are that they can operate at relatively high frequencies, and they observe good linear behaviour and a moderate hysteresis between 2% [15]. They can operate at comparatively higher temperatures than piezoelectric and electro-strictive materials. A disadvantage is that a magnetic field is needed to control the material; therefore, they are not easily embedded in control schemes. The most well-known magneto-strictive material is TerFeNol-D (Terbium Iron).

2.1.4. Electro-strictive

Electro-strictive materials strain proportionally to the square of an applied electric field, and unlike piezoelectric materials, they are not poled. They can strain up to 0.2%, present a low hysteresis, but due to the quadratic response to an electric field, they are highly non-linear and very sensitive to temperature variations. Lead Magnesium Niobate (PMN) and Polyvinylidene Fluoride (PVDF) are the most well-known electro-strictive materials.

2.1.5. Magneto/Electro-rheological (MR/ER)

These materials are mainly fluids that can experience change in their rheological properties (plasticity, elasticity, viscosity and yield stress) when an electric or magnetic field is applied; once the stimulation is removed, their original rheological properties are restored. These fluids are a combination of some kind of oil mixed with micro-particles (dielectric, metallic or polymeric), which are the ones that polarize themselves when a field is applied. A difference between the ER and MR is that ER require a high voltage and MR require a high current to operate, and that ER are more sensitive to impurities in the fluid. These materials are being developed for use in car shocks, damping washing machine vibration, prosthetic limbs, exercise equipment, clutches, valves and engine mounts to reduce noise and vibrations in vehicles.
2.1.6. Comparison between Smart Materials

A comparison chart (Table 2) of the past mentioned smart materials was built to compare their principal characteristics, by which an engineer or researcher might decide to select.

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>Product</th>
<th>Strain (%)</th>
<th>Frequency</th>
<th>Max. Temp (°C)</th>
<th>Density (g/cm³)</th>
<th>Max. Force</th>
<th>Hysteresis (%)</th>
<th>Control Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezoelectric</td>
<td>PZT</td>
<td>0.1 - 0.2</td>
<td>GHz</td>
<td>300</td>
<td>7.5 - 7.8</td>
<td>High</td>
<td>15 - 20</td>
<td>Voltage/Current</td>
</tr>
<tr>
<td>Magnetostrictive</td>
<td>Terfisol-D</td>
<td>0.1 - 0.5</td>
<td>KHz</td>
<td>400</td>
<td>9</td>
<td>Medium</td>
<td>1 - 2</td>
<td>Magnetic Field</td>
</tr>
<tr>
<td>Electrostrictive</td>
<td>PMN, PVDF</td>
<td>0.1 - 0.2</td>
<td>KHz</td>
<td>300</td>
<td>7.8</td>
<td>High</td>
<td>0 - 1</td>
<td>Voltage</td>
</tr>
<tr>
<td>SMA</td>
<td>NiTiNi</td>
<td>4 - 5</td>
<td>Hz.</td>
<td>300</td>
<td>7</td>
<td>Medium</td>
<td>5 - 40</td>
<td>Temp./Current</td>
</tr>
<tr>
<td>MR fluid</td>
<td>-</td>
<td>-</td>
<td>KHz</td>
<td>150</td>
<td>3.4</td>
<td>-</td>
<td>-</td>
<td>Magnetic field</td>
</tr>
<tr>
<td>ER fluid</td>
<td>-</td>
<td>-</td>
<td>KHz</td>
<td>125</td>
<td>4.2</td>
<td>-</td>
<td>-</td>
<td>Voltage</td>
</tr>
</tbody>
</table>

Table 2 - Smart materials comparison

2.1.7. Other types of Smart Materials

Other kinds of smart materials [26] are:

- **Magnetic shape memory alloys**: materials that change their shape in response to a significant change in the magnetic field.

- **pH-sensitive polymers**: materials which swell/collapse when the pH of the surrounding media changes.

- **Halo-chromic materials**: materials that change their colour as a result of changing acidity.

- **Chromogenic materials**: they change colour in response to electrical, optical or thermal changes. These include electro-chromic materials, which change colour or opacity on the application of a voltage (e.g., liquid crystal displays), thermo-chromic materials change in colour depending on their temperature, and photo-chromic materials, which change colour in response to light (e.g., light sensitive sunglasses).

- **Non-Newtonian fluid**: liquid which changes its viscosity in response to an applied shear rate.
• **Elasto-strictive materials**: these materials are the mechanical equivalent to electro/magneto-strictive materials. They exhibit a high hysteresis between stress and strain.

• **Thermo-responsive materials**: amorphous and semi-crystalline thermoplastic polymeric materials that suffer changes in their specific volume of polymers at their glass transition temperature.

• **pH-sensitive materials**: materials that change their colour as a function of pH, and are also reversible.

• **Smart Polymers**: polymeric systems that are capable of responding strongly to slight changes in the external medium. Some properties that can vary are volume, coefficient of thermal expansion, specific heat, heat conductivity, modulus and permeation.

• **Smart Gels (Hydro-gels)**: a combination of the concept of solvent-swollen polymer networks in conjunction with the material being able to respond to other types of stimuli like temperature, pH, chemicals, pressure, stress, light intensity, radiation.

### 2.2. Smart Structures

Two paradigms exist on the definition of a Smart Structure [25]: The scientific paradigm, which describes a smart structure as a structural system with a macrostructure, or maybe microstructure, with “intelligence” and “life” features integrated, so to provide environmental adaptive functionality. On the other hand, the technological paradigm, the one of interest for this research, defines a smart structure as the integration of a mechanical structure with sensors, actuators and controls, to accomplish a specific purpose [27]. Figure 1 presents a model [25] for this paradigm:
As smart materials can be used as sensors or actuators, a smart structure might contain one or more of those. Smart materials can be bonded into a structural component via surface adhesion, incrustation, embedding or encapsulation. The use feasibility in these kinds of advanced system structures has gained interest for different reasons: low energy consumption, no moving parts, high reliability, weight reduction, and a large variety of materials with different properties exists, so they can be adapted to particular purposes.

With the continuous development of smart materials and structures, one can imagine a wide range of possibilities [10]. Engineering structures could operate at limit conditions without fear of exceeding them with the help of a structural modification control. Moreover, a full maintenance report, including performance history and location of irregularities, could be generated for maintenance purposes, therefore preventing sudden failure.

Currently, R&D of these materials and structures are mainly focused in industries such as aerospace, automotive, civil engineering and medical industries [1-12]. In the field of aerospace, research is carried out in areas such as flexible wings modification to control the aero-elastic shape, or structural Health and Usage Monitoring Systems (HUMS). The automotive relies heavily on smart materials, such as air-bag sensors or ABS and active road control systems (i.e. active suspensions). Civil engineers are also trying to implement HUMS systems, but they also focus on reducing vibrations in structures (i.e. bridges, dams, skyscrapers). Bio-compatible smart materials such as SMA are used in the medical industry to develop bone plates, but also in the bio-technology sector to develop different kinds of sensors or robotic applications. It is clear that there is a great potential for these devices in a variety of applications in the near future.
Given their fast response, piezoelectric materials have proven useful in applications involving vibration reduction of mechanical structures using different control approaches [9, 28-32]. High induced forces, relatively good linearity and easy of access to controlling equipment are aspects considered when selecting piezoelectric materials.

2.3. Piezoelectricity and Piezoceramics

This section presents a brief history of piezoelectricity, some of the fundamentals of piezoelectricity, and general information about piezoceramic actuators.

2.3.1. History of Piezoelectricity

In 1880, the first scientific publication that described piezoelectricity was published by the brothers Pierre and Jacques Curie. They were conducting a variety of experiments on a range of crystals that displayed surface charges when they were mechanically stressed, demonstrating the direct piezoelectric effect. However, they did not predict the converse piezoelectric effect. It was rather deduced mathematically from fundamental principles of thermodynamics by Gabriel Lippmann in 1881. After this, the Curies confirmed experimentally the existence of the converse effect in piezoelectric crystals.

For the next few decades, piezoelectricity generated significant interest within the European scientific community, and continued to do so until World War I, when a first practical application was developed, an ultrasonic submarine detector: the sonar. It was developed in France in 1917 by Paul Langevin and co-workers, and it consisted on a transducer made of a mosaic of thin quartz crystals that was glued between two steel plates, and a hydrophone to detect the returned echo. The device was used to transmit a high-frequency chirp signal into the water, and then to measure the depth or distance to an object by timing the return echo.

Between the two World Wars, piezoelectric crystals were employed in many applications such as frequency stabilizers for vacuum-tube oscillators, ultrasonic transducers used for measurement of material properties, and many commercial applications were developed, such as microphones, accelerometers, phonograph cartridges and ultrasonic transducers.

During World War II, research groups in the United States, Japan and Russia developed a new class of man-made materials with very high dielectric constants. Piezoceramic
Chapter 2: Literature Survey

materials such as barium strontium titanate (BaSTO) and lead zirconate titanate (PZT) were discovered as a result of these activities, and a number of methods for their high-volume manufacturing were devised. The ability to build new piezoelectric devices by tailoring a material to a specific application resulted in a number of developments and inventions such as piezo ignition systems or sensitive hydrophones.

2.3.2. Fundamentals of Piezoelectricity

Piezoelectric materials exist primarily in two forms: ceramic and polymer. The primary use of ceramics is as actuators, and the most common ceramics are PZT and BaSTO. Polymer piezoelectrics, in the other hand, are better used as sensors, such as polyvinylidene fluoride (PVDF). Before poling, piezoelectric materials are isotropic, and once polarized, they behave anisotropic in a micro sense but transversely isotropic in a macro sense [25].

A piezoelectric ceramic is a mass of perovskite crystals [33]. Each crystal is composed of a small, tetravalent metal ion placed inside a lattice of larger divalent metal ions and $O^2$, as shown in the next figure:

Above a critical temperature, known as Curie temperature, each perovskite crystal in the heated ceramic element exhibits a simple cubic symmetry with no dipole moment; however, at temperatures below the Curie temperature each crystal has tetragonal symmetry and a dipole moment. Adjoining dipoles form regions of local alignment called Weiss domains, which gives a net dipole moment to the domain, and thus a net polarization. As shown in Figure 2, the polarization direction among neighbouring domains is random, and the ceramic has no overall polarization [33].
The poling process consists on applying a strong DC electric field to the element, usually at a temperature slightly below the Curie temperature, causing the domains to align (Figure 3b). After cooling, the domains nearly stay in alignment, presenting a remnant polarization (Figure 3c), which can be degraded by exceeding the mechanical, electrical and thermal limits of the material.

When a subsequent electric field is applied to the poled piezoelectric material, the Weiss domains increase their alignment proportional to the field, and result in a change of dimensions (compression or extension) of the material as shown in Figure 4(d-e). Compression along the direction of polarization, or tension perpendicular to the direction of polarization, generates voltage of the same polarity as the poling voltage (Figure 4b), and an inverse force will generate a voltage with polarity opposite to that of the poling voltage (Figure 4c).

2.3.3. Piezoceramic Actuators

Depending on the poling process and configuration of the piezoelectric material, piezoceramic actuators act in different modes as shown in Figure 5. The most widely used piezoceramics are manufactured in thin sheets, which can later be easily embedded to a structure. The basic modes of action are the transverse and longitudinal motor modes. Both of them are excited through the thickness of the material, but the transverse motor
(\textit{d}_{31} \ mode) acts along the four thin sides, while the longitudinal motor (\textit{d}_{33} \ mode) acts along the wide surfaces.

Bi-morphs (double-layer) configurations are also available commercially [25]. They consist of two layers of piezoceramic material stacked with a thin shim (typically brass) between them. If the two sheets are poled in the same direction, the actuator will act in the compression/extension mode, providing twice the force; and when an opposite polarity is applied to the sheets, a bending action is obtained (one sheet expands and one sheet contracts). Stacking of layers is also possible, where multi-layers are stacked on top of one another, always with opposite poling. Top and bottom of each layer are alternatively connected to the voltage terminals. This configuration provides a much greater force than every other mode in the poling axis.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{piezoceramic_actuators_modes.png}
\caption{Piezoceramic actuators modes}
\end{figure}

In various cases, actuators are composed of a packaged device including the piezoceramic material and other composite material layers. The function of these layers is to make the actuator less fragile and easier to handle, as well as to prepare the electrical connections for the user. There are several commercial manufacturers of piezoceramic actuators, as well as sensors, such as: PI Ceramic, Piezo Systems Inc., Morgan Matroc, Channel Industries, EDO Corp., Staveley Sensors Inc., MIDE, Thunder, Sensor Tech. Ltd., APC International Ltd., CEDRAT, DSM, Ferroperm, Trek, Boston Piezo-Optics Inc.
2.4. Piezoceramic Actuators: Research and Applications

One of the main focuses of researchers has been the use of piezoelectric materials in vibration control and suppression in the automotive, aerospace and structural ambits [1-12]. Bein et al. [1] used a semi-active electromechanical vibration absorber based on a piezoelectric patch actuator to reduce vibrations of a structure, focused on automotive applications. Vibration reduction on automotive shafts was investigated by Kunze et al. [2]. Jalili, Wagner & Dadfarnia [3] investigated the design of an innovative piezoelectric ceramic based actuator mechanism with a stepping motion amplifier to deliver force and displacement at higher magnitudes and operating frequencies, for an engine valve train application.


Structural health monitoring is also of research importance, such as for Mayer et al. [34], who examined an approach for model based monitoring of piezoelectric actuators. Strassberger & Waller [8] used structural control for reduction of sound radiation using piezoelectric actuators. Sloss et al. [9] studied the effect of axial force in the vibration control of beams by means of an integral equation formulation.

Passive vibration control has also been discussed for a couple of decades, and it has grown in popularity as new methodologies for their use and new applications have been established, such as vibration control in tennis rackets or water skis [25].

Belouettar et al. [35] focused on nonlinear vibrations, due to geometric nonlinearity and piezoelectric effects, on a combination of piezoelectric-elastic-piezoelectric sandwich beams submitted to active control. Gao & Shen [36] also investigated geometrically non-linear transient vibration response and control of plates with piezoelectric patches subjected to pulse loads.

Other research lines are: active structural acoustic control, shape control of surfaces and flow control of fluids.
2.5. Piezoelectric and Hysteresis Modelling

In the past years, several models have been proposed by various authors for piezoelectric materials which account principally for hysteresis. Some of the proposed models include some dynamic behaviour and others are used to develop a control scheme for piezoceramic actuators.

The mostly used hysteresis model is the Classical Preisach Model (CPM) [37-42]. The model is shown to offer excellent modelling accuracy when the actuator is not subjected to any loads, excited by a low frequency voltage signal [37]. A Preisach-type hysteresis, a feed-forward controller and a PD-type feedback controller was used for positioning control by Jang, Chen & Lee [38]. Yu [39] proposed a new Preisach model and a new approach with Wavelet identification. Applications such as the use of a piezo-stack actuator to move a trailing-edge flap for helicopter vibration control was researched by Viswamurthy & Ganguli [40] using the Preisach model.

Other researchers use the Generalized Maxwell Slip (GMS) model [22, 23, 43-48], also known as the Maxwell Resistive Capacitive (MRC) model, which is said to be a subset of the more general Preisach hysteresis model [22]. This model has better correspondence with the results of the physically motivated friction model in the case of frictional lag and transitional behaviour, without adding extra parameters in the model compared to existing models [43]. Goldfarb & Celanovic [23] proposed the MRC model as a lumped-parameter casual representation of the rate-independent hysteresis. An electro-mechanical model was also considered, as well as a connection between the two domains. Georgiou & Mrad [22] presented a similar model that characterizes hysteresis based on the GMS model and describes both the electrical and mechanical properties of piezoceramics, with the difference of having two electromechanical coupling values and a charged-limited resistance. Lee [44] used the GMS model and presented an inverse model for hysteresis compensation. A Proportional Integral Derivative (PID) controller together with a GSM model was presented by Choi, Oh & Choi [45]. The GMS model was compared to the LuGre model and Leuven model by Lampaert, Al-Bender & Swevers [43]. Huang & Lin [49] also compared the GMS model to the Bouc-Wen and Duhem models. A Dynamic NonLinear Regression with direct application of eXcitation (DNLRX) method was presented by Rizos & Fassois [46] to identificate the GMS model. Wood, Steltz & Fearing [48] used the GSM model for hysteresis, together with a Kelvin-Voigt model for creep.
Richter et al. [50] presented a nonlinear model that encompasses creep, nonlinear voltage dependence, and hysteresis (using a Voigt unit), for the development of high precision piezoelectric tube actuators.

Deng & Tan [51] presented a non-linear moving average model with exogenous inputs (NMAX) and a non-linear auto-regressive moving average model with exogenous inputs (NARMAX) to model static and dynamic hysteresis. It has the advantage of a systematic design procedure which can update on-line the model parameters so as to accommodate to the change of operation environment compared with the Preisach model. Another model is the Prandtl-Ishlinskii (P-I) used by [21, 41, 52]. This P-I model is based on a rate-independent backlash operator [21]. Najafabadi et al. [21] proposed an adaptive inverse control method based on a modified PI operator, which compensates both the rate dependent hysteresis nonlinearity and the mechanical loading effect. Shen et al. [52] modified the P-I model and proposed a sliding-mode controller to compensate the remaining nonlinear disturbances. One advantage of the P-I model over the CPM model, according to the author, is that it is less complicated and that its inverse can be computed analytically, although it is less accurate. Changhai & Ling [53] described a method for simultaneous compensation of the hysteresis and creep of piezoelectric actuator based on an inverse control in open-loop operation. Creep was also of main interest to Yeh, Ruo-Feng & Shin-Wen [54] and Richter et al. [50], who also modelled hysteresis based on a Four-Element Burgers model together with a Voigt element. More recent hysteresis models include Neural Networks as presented by Dang & Tan [55]. Ha, Fung & Yang [56] used a Leuven model of the frictional force to modify dynamic equations and an adaptive identification method to experimentally identify the hysteresis parameters of the Bouc-Wen model. Royston et al. [18] characterized theoretically and experimentally the nonlinear behaviour of a 1-3 piezoceramic composite. They analyzed how quasi-static and dynamic mechanical response phases to harmonic electrical excitation over a range of excitation frequencies and two different mechanical loading conditions.

Damjanovic [57] explained the open-loop inverse model hysteresis reduction, by having an actuator’s input-output relation inverse map, so a new input signal can be calculated from the model. Tzen, Jeng & Chieng [58] combined the second order model with a cascaded hysteresis non-linearity for a piezoelectric actuator, and proposed an inverse model which involves exponential lag.
Summarizing, there are three principal hysteresis models: the CPM, GMS and P-I. The CPM is shown to offer excellent modelling accuracy when the actuator is not subject to any load and is subject to an excitation voltage signal at a low frequency [37]. It uses first order recursive curves to approximate the hysteresis nonlinearity. It has the disadvantage of using a large experimental database and having a time consuming parameter estimation procedure. Also, the CPM model needs to spend much time on computation during the control process [21]. Being less accurate than the CPM model but at the same time less complex [52], the P-I model has unique properties that are invertible [41], and an inverse model, used to reduce hysteresis nonlinearity, can be computed analytically. The GMS model is a subset of the more general Preisach hysteresis, and it has the advantage that parameterization can be achieved in one simple experiment [22]. It has a good interpretation and does not require a priori knowledge of the system's physical parameters [46].
Chapter 3

Piezoceramic Actuator (PA) Modelling

(As from this point, the term “piezoceramic actuator” will be abbreviated to PA).

For the purpose of this investigation, a PA was selected as object of this research. Due to their fast response and wide range operational bandwidth, as well as their controlling capabilities, these smart materials are ideal for many applications. On the other hand, undesired behaviour of these devices, such as nonlinearities and hysteresis, need to be compensated for precise control. Thus, a characterization and modelling process needs to be developed.

3.1. Piezoelectric Constitutive Equations

Piezoelectric materials operate under two effects: the converse effect, when it undergoes a strain or mechanical deformation in response to an applied electrical field; and the direct effect, when an electrical charge is produced when it comes in contact with an applied stress.

Common denominations in the axes of a piezoceramic element are identified by numbers rather than letters. Generally, axis-3 corresponds to the $z$ axis and is assigned to the direction of the initial polarization of the piezoceramic, while axis-1 or $x$, and axis-2 or $y$ lie in the plane perpendicular to axis-3.

![Piezoelectric axis nomenclature](image)

IRE (Institute of Radio Engineers), which later became IEEE (Institute of Electrical and Electronics Engineers), have developed a series of documents [19, 20] regarding the standards on piezoelectric crystals since 1949. The last IEEE document in this field, also
approved by the American National Standards Institute (ANSI), stated the Standard on Piezoelectricity (refer to Appendix A for more information). From this document, the linear constitutive equations can be obtained. The primarily equations used for acting are:

Converse effect

\[ S_i = s^{E}_{ij}T_j + d_{mi}E_m \]  \hspace{1cm} (1)

Direct effect

\[ D_m = d_{mi}T_i + \varepsilon^T_{mk}E_k \]  \hspace{1cm} (2)

Alternative formulations, mainly used for sensing, are:

Converse effect

\[ S_i = s^{D}_{ij}T_j + g_{mi}D_m \]  \hspace{1cm} (3)

Direct effect

\[ E_m = -g_{mi}T_i + \beta^T_{mk}D_k \]  \hspace{1cm} (4)

Other representations of the constitutive equations, depending on the components taken as independent variables are:

\[ T_i = \varepsilon^{E}_{ij}S_j - e_{mi}E_m \]  \hspace{1cm} (5)

\[ D_m = \varepsilon_{mi}S_i + \varepsilon^S_{mk}E_k \]  \hspace{1cm} (6)

\[ T_i = \varepsilon^{D}_{ij}S_j - h_{mi}D_m \]  \hspace{1cm} (7)

\[ E_m = -h_{mi}S_i + \beta^S_{mk}D_k \]  \hspace{1cm} (8)

Where the indexes \( i,j = 1,2, ..., 6 \) and \( m,k = 1,2,3 \) refer to the different directions within the material coordinate system as shown in Figure 6. Also, the superscript “E” is used to state that the elastic compliance \( s^{E}_{ij} \) is measured with the electrodes short-circuited (meaning a zero or constant electric field); the superscript “D” in \( s^{D}_{ij} \) denotes that the measurements were taken when the electrodes were left open-circuited (meaning zero or constant electric displacement); and the superscripts “T” and “S” denote that the measurements where taken at zero or constant stress or strain respectively.

If we assume the device is poled along the axis-3 and assuming transversely isotropic properties (the case of piezoceramics), some parameters of the matrices in equation (1) to (8) will become zero or will be expressed in terms of other parameters [20, 25], for example:
\[ s_{11} = s_{22} \]  
\[ s_{13} = s_{31} = s_{23} = s_{32} \]  
\[ s_{12} = s_{21} \]  
\[ s_{44} = s_{55} \]  
\[ s_{66} = 2(s_{11} - s_{12}) \]  
\[ d_{31} = d_{32} \]  
\[ d_{15} = d_{24} \]  
\[ \varepsilon_{11} = \varepsilon_{22} \]  

In the end, simplified matrixes (i.e. equation (1) and (2)) are obtained:

\[
\begin{bmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6
\end{bmatrix}
= \begin{bmatrix}
s_{11} & s_{12} & s_{13} \\
s_{12} & s_{11} & s_{13} \\
s_{13} & s_{13} & s_{33} \\
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6
\end{bmatrix}
+ \begin{bmatrix}
d_{31} \\
d_{31} \\
d_{33} \\
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\]  

\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix}
= \begin{bmatrix}
d_{31} & d_{31} & d_{15} \\
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{11} \\
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\]  

where all coefficients not shown are zero. Considering a PA of length \(l_p\), width \(b_p\), and thickness \(t_p\), two main concepts need to be described [16, 17]. If the actuator is in a free position (not attached to any structure) and an electric field (V/m) is applied to the polarization axis, the actuator will strain in all three axes according to its piezoelectric strain constants. For example, if a field is applied to the axis-3 and no stress is acting on the material, the free strain (\(\Lambda\)) in axis 1 can be calculated according to equation (17):

\[ S_1 = s_{11} \overrightarrow{T_1} + s_{12} \overrightarrow{T_2} + s_{13} \overrightarrow{T_3} + d_{31} E_3 \]  

21
In a similar way, if a PA is constrained so that it cannot deflect in one of its axes, and an electric field is applied, a blocked force \( F_b \) is generated. Taking the same last example, but constraining the actuator in axis-1, the resulting force in axis-1, as given by equation (19), would be:

\[
F_b = -Y_{11}d_{31}b_p V
\]  \( \text{(22)} \)

Where, \( Y_{11} \) is the elastic modulus along axis-1, and \( V \) is the applied voltage. For a constant voltage these two values can be plotted, and a line joining them represents the force-strain acting range that the PA will follow.

3.2. Non-linear behaviour of Piezoceramic Actuators

Previously mentioned constitutive equations use linear coefficients, but when accuracy is paramount, non-linear behaviour must be taken into account. Piezoelectric materials possess several non-linear characteristics, such as: material and piezoelectric nonlinearities, dynamic behaviour, and hysteresis.
3.2.1. Hysteresis

Hysteresis is a nonlinear phenomenon that occurs when a small mechanical strain remains in the piezoelectric material upon removal of the electric field. This is an electrical property that piezoelectric materials possess, which mainly exists between the applied electrical field and the resulting electrical charge. Some theories explain hysteresis as caused by the dissipation of energy due to sliding events in the polycrystalline piezoelectric body.

It has been demonstrated that by controlling the electrical charge or current, the hysteresis effect can be considerably reduced [33]. But since charge control is more complex in practice, some techniques have been developed with the purpose of reducing hysteresis in voltage-driven PAs. A few examples are phase control and inversion-based models. [33]

![Figure 8 - Typical piezoelectric voltage vs. charge hysteresis](image-url)

3.2.2. Dynamic behaviour

PA dynamic behaviour can be considered as a second order linear dynamic model [21, 23, 46, 56, 58]. Therefore, frequency response needs to be characterized to prevent operation at resonant frequencies.

3.2.3. Material and Piezoelectric Nonlinearities

For relatively large applied electrical fields or forces, non-linear variations occur and a polynomial curve fits better for singular coefficients [25]. For example the piezoelectric coefficient $d_{31}$, which varies as a function of electric field as showed in the next figure:
The stress-strain relationship, the elastic compliance coefficient, also presents a well-known non-linear behaviour [25]. Below the elastic limit, the ratio remains constant, but above the elastic limit, it will vary until the ultimate strength point is reached.

An interesting behaviour in piezoelectric materials shows that since a mechanical stress causes an electrical response, which in turn can increase the resultant strain, the effective Young’s modulus with the electrodes being short-circuited ($\varepsilon^{P}_{ij}$) will be smaller than the modulus of elasticity when it is open-circuited ($\varepsilon_{ij}$) [33].
3.3. Piezoceramic Actuator Modelling

Theoretical foundation for the modelling of the PA will now be explained. Each considered property of the PA is based on a specific model, which will be included in the final complete model.

3.3.1. Hysteresis based on the Generalized Maxwell Slip (GMS) model

The Generalized Maxwell Slip (GMS) model can be considered as a subset of the more general Preisach operator characterized by specific properties that facilitate the identification process [22]. Its general form allows applying the model to different cases, such as: the stress-strain relation in elasto-plastic materials, magnetic field-flux density in highly magnetic materials, voltage-charge relation in piezoelectric materials, or temperature-entropy relation, among others [23].

To understand the concept, a mechanical formulation is first proposed. The behaviour can be modelled by combining an ideal spring which represents a pure energy-storage, coupled to a pure Coulomb friction element, representing a rate-independent dissipation [23]. A representation form is presented in Figure 11, where a massless block subjected to Coulomb friction is joined to a massless linear spring, and an external force is applied to the system.

![Figure 11 - Single elasto-slide element](image)

With reference to Figure 11 and Figure 12: \( f \) is the breakaway friction force of the block, \( \mu \) is a friction coefficient, \( N \) is a normal force acting on the block, \( F \) is the reaction force, \( k \) is the linear stiffness of the spring, \( x \) is an external displacement input and \( x_b \) is the current position of the block. When a displacement is input, a linearly increasing reaction force will be sensed (see Figure 12 (a-b)), until the force reaches the static friction limit of the block (b). From this point onwards, the whole element, including the block, will slide (b-c) in a dynamic condition. The whole static-dynamic interaction will present a hysteretic behaviour represented by equations (23) and (24):
\[ f = \mu N \]  

\[
F = \begin{cases} 
  k(x - x_b) & \text{if } |k(x - x_b)| < f \\
  f \text{ sgn}(\dot{x}) \text{ and } x_b = x - \frac{f}{k} \text{ sgn}(\dot{x}) & \text{else}
\end{cases}
\]  

Figure 12 - Single elasto-slide element behaviour

Now, if a set of elasto-slide elements are put in parallel, each having a different breakaway force, a new behaviour is obtained as shown next:

Figure 13 - Multiple elasto-slide elements behaviour

The constitutive formulation for this case, considering \( n \) elasto-slide elements, is defined by equations (25) and (26), where \( f_i, F_i, k_i \) and \( x_{b_i} \) are the breakaway friction force, output reaction force, spring linear stiffness and block position, respectively, of the \( i^{th} \) elasto-slide element.
To model this rate-independent hysteresis, it requires the parameterization of the initial rising curve of the hysteresis from a relaxed state, as shown in Figure 13. For this, the curve can be divided in \( n \) segments, each \( j^{th} \) segment having a different slope \( s \). Therefore, to build a curve fit, only \( 2n \) values are needed, the slope \( (s_j) \) and the location \( (x_j) \) of each segment, each one defined as:

\[
F = \sum_{i=1}^{n} F_i
\]

\[
F_i = \begin{cases} 
  k_i(x - x_{b_i}) \text{ if } |k_i(x - x_{b_i})| < f_i \\
  f_i \operatorname{sgn}(\dot{x}) \text{ and } x_{b_i} = x - \frac{f_i}{k_i}\operatorname{sgn}(\dot{x}) \text{ else}
\end{cases}
\]

It is important to mention that since the Maxwell slip model is a linear approximation, the accuracy of the model will increase if the number of segments increases. Having mentioned before that this particular model is not domain-specific, this mechanical representation can also represent the rate-independent hysteretic relationship between voltage and charge in a piezoelectric material. Some modifications and relationships (equations (29) and (30)) between the mechanical and now electrical model are made to equations (25) and (26), finally resulting in equations (31) and (32), where \( v_i, V_i, C_i \) and \( q_{bi} \) are the breakaway voltage, the output voltage, capacitance value and charge level, respectively, for each \( i^{th} \) element; \( V_H \) is the hysteresis voltage.

\[
v_i = f_i
\]

\[
\frac{1}{C_i} = k_i
\]

\[
V_i = \begin{cases} 
  \frac{(q - q_{bi})}{c_i} \text{ if } \left| \frac{(q - q_{bi})}{c_i} \right| < v_i \\
  v_i \operatorname{sgn}(i) \text{ and } q_{bi} = q - C_i v_i \operatorname{sgn}(i) \text{ else}
\end{cases}
\]

\[
V_H = \sum_{i=1}^{n} V_i
\]
3.3.2. Electro-mechanical Dynamic model

A set of linear constitutive equations were presented previously, but as the development of this investigation proceeded, it was realized that modifications needed to be done for the validation of a more realistic model. Recapitulating equations (1) to (8) for the direct and converse effect of the piezoceramic, a clear electrical-mechanical relation exists, as the diagram in Figure 14 shows.

A new electro-mechanical model based on previous research [22, 23] is presented. It accounts for the dynamic behaviour due to the frequency response of the actuator, the voltage-charge hysteresis present in piezoelectric materials, and the non-linear coefficients that exists in the material and piezoelectric properties. The model diagram is presented in Figure 15.

![Figure 14 - Electrical and mechanical relations in piezoelectrics](image)

![Figure 15 - Electro-mechanical model representation](image)

The electrical input to the model is the voltage across the PA denoted by $V_{in}$. $V_H$ represents the hysteresis voltage from the Maxwell slip model previously presented, as a function of
the total input charge \( q \) or current \( \dot{q} \), and \( V_T \) is the voltage across the linear capacitance \( C \) of the actuator’s material. \( F_{ext} \) corresponds to an external force input to the actuator in the mechanical domain, and the dynamic behaviour is presented as a second order model in \( m, k \) and \( b \), the mass, stiffness and damping of the actuator, respectively. \( x \) is the output displacement of the actuator, and \( n_{em} \) represents the electro-mechanical coupling in the model, which transforms the transformer voltage \( V_T \) into an input force \( F_T \) in the mechanical side (or for a direct effect, transforms the output displacement into a charge \( q_T \) or current, in the electrical side). Note that the electro-mechanical couple and stiffness are non-linear, represented as a function of the input voltage and displacement, respectively. This electro-mechanical model is therefore described by:

\[
V_{in} = V_H + V_T \tag{33}
\]

\[
V_H = f(q) \tag{34}
\]

\[
n_{em} = f(V_{in}) \tag{35}
\]

\[
\dot{q} = \dot{q}_C + \dot{q}_T = \dot{V}_TC + n_{em}\dot{x} \tag{36}
\]

\[
F_T + F_{ext} = m\ddot{x} + b\dot{x} + kx \tag{37}
\]

\[
F_T = n_{em}V_T \tag{38}
\]

\[
k = f(x) \tag{39}
\]

### 3.3.3. Non-linear coefficients modelling

Non-linear behaviour is expected in the piezoceramic as well as the elastic compliance coefficients. Their values ought to be dependent on an input stimulus, such as voltage, strain or stress.

In Figure 14, it is observed that a relationship exists between the coupling factor \( n_{em} \) (see electro-mechanical model of Figure 15) and the inverse piezoelectric voltage coefficient \( e \). It relates the Electric Field (or Voltage \( V_T \)) to the Stress generated (or Force \( F_T \)), and also the Strain (or Displacement \( x \)) to the Electric Displacement (or Charge \( q_T \)). This coefficient is defined by [59]:

29
\[ e_{31} = \varepsilon_0 K_3^T / d_{31} \]  \hspace{1cm} (40)

Where \( \varepsilon_0 \) is the permittivity of free space with a value of \( 8.85 \times 10^{-12} \text{ C/Vm} \) and \( K_3^T \) is a relative dielectric constant [19]. Since \( d_{31} \) represents a non-linear function dependent on the input voltage, \( e_{31} \) and \( n_{em} \) will also be non-linear functions. Finally the electro-mechanical couple for the model can then be defined by:

\[ n_{em} = b_p / e_{31} \]  \hspace{1cm} (41)

It can also be seen in Figure 14b that the elastic compliance coefficient (\( s_{ij} \)) is related to the stiffness value (\( k \)) in Figure 15. Both relate the input force (or stress) to the output displacement (or strain), therefore a polynomial function depending on the total displacement (or strain) would better fit the stiffness value, if found non-linear.
Chapter 4

Experimental Set-up

4.1. Characterization Experiments

Several characterization procedures \[19, 20, 59\] have been published for piezoelectric materials, each for a different purpose. Some of those are: the resonant method and the direct methods, which involve quasi-static and dynamic measurement procedures.

4.1.1. The resonant method

This method relies on the fact that a given body resonates at specific frequencies. When the body is excited at one of these \(f_r\), the body will resonate freely with a comparatively greater amplitude; at any other frequency, the body will oscillate at smaller amplitudes, having a maximum impedance at its anti-resonant frequency \(f_a\) \[19, 20, 59\].

Measuring these frequencies for different modes of vibration (longitudinal, transversal and out-of-plane), would give information about the elastic properties associated with that mode.

One important consideration for applying this method is that adjacent modes must be attenuated, in order to obtain reliable results. For this end, a sample geometry is chosen for measuring piezoelectric and elastic coefficients, as shown in Figure 16. This method is only suitable for piezoelectric materials in their crystalline form, (i.e. not an embedded PA).
4.1.2. Direct methods

They are used to quantify the direct and converse effects in ceramic samples [59] as well as to explore ceramic’s behaviour regarding hysteresis, nonlinearities, frequency response, aging, thermal behaviour and creep [59]. These methods are more suitable for materials with the configuration of an actuator or a sensor.

4.1.2.1. Quasi-static measurements

These measurements are carried out at frequencies much lower than the fundamental elastic resonance. Typically, a known input (i.e. force or electric field) is applied to the ceramic, and the corresponding output (strain or charge) is measured and recorded.

For each measured property, different constraints are applied (such as zero or constant strain, stress, voltage, charge); therefore, a complex test bench might be needed.

4.1.2.2. Dynamic measurements

For these measurements, typically a sine sweep is involved measuring frequency response and phase lag. A sine sweep input excitation and measurement of amplitude and phase variations is typical.

When performing dynamic measurements, the power consumption increases as a function of frequency and needs to be considered for optimal operation of the piezoelectric material.
and equipment. The power required to drive a piezoelectric ceramic [60] can be calculated from the following equation:

\[ P = \pi f C V^2, \]

where \( P \) is the power in Watts, \( f \) is the operating frequency in Hz, \( C \) is the piezoelectric capacitance in Farads, and \( V \) is the operating maximum voltage.

4.2. Equipment

The equipment needed for the experimental characterization of PAs is listed next:

- **Voltage amplifier**: Because PAs need a high voltage input range to operate, a device that can amplify the variable input control signal is needed. For this case, a TREK model PZD 700 amplifier which can deliver up to \( \pm 700V \) was used.

![Figure 17 - TREK PZD 700 voltage amplifier](image)

- **Control signal generator & Data acquisition system**: Whether a simple signal generator or a controlled signal generator, a special device is required. For these experiments, a dSPACE DS1104 R&D Controller Board with a CP1104 Connector Led Panel was used. By using Matlab/Simulink® and linking it with a dSPACE RTI interface, several kinds of signals can be generated. In this way, if the PA is attached to a mechanical system with a closed control loop, a control signal could easily be delivered to the PA.

For the data to be reliable, it must be measured in real-time at a frequency higher than the lowest operating frequency of the system. All the sensor signals were acquired via an A/D converter from the same dSPACE Controller Board, and paired with the controlled input signal to obtain data charts from Matlab/Simulink.
Chapter 4: Experimental Set-up

Figure 18 - dSPACE DS1104 connector LED panel

- **Strain sensor**: Used to measure strain in different axis of the actuator. A rectangular rosette of precision strain gages from Measurements Group, Inc. type CEA-06-250UR-120 (Figure 19) was installed on the actuator to measure strain in x and y axis, and shear strain $xy (\varepsilon_x, \varepsilon_y, \gamma_{xy})$. For this, a strain gage indicator from Vishay Model P-3500, with a quarter Wheatstone bridge (Figure 20), was also used.

Figure 19 - Rectangular strain gage rosette

Figure 20 - Vishay P-3500 portable strain indicator
• **Force sensor:** Some characterization techniques require measuring the applied (exerted) force/stress on (by) the piezoceramic. In this case, a load cell type was selected: manufactured by CNCELL and model number PA6110, with a capacity of 25 lbf (see Appendix C for more information).

![Figure 21 - CNCell PA6110 load cell](image1)

• **Test Bench:** A test bench for carrying out some of the experiments was designed, manufactured and assembled, as seen in Figure 22 (design dimensions can be found in Appendix E). The objective of this test bench is to induce a controlled stress in the axis-1 of the PA, while leaving other axes unconstrained.

![Figure 22 - Test bench for stress induction](image2)
4.3. Set-up for measurements

Before performing any measurement, a strain gage rosette was surface-bonded to the actuator under study, and all the equipment presented in the last section was interconnected together as shown in Figure 24. Each one of the sensors and actuators were connected to the dSPACE Connector Panel, which was connected to the Controller Board inside a PC. Functionality and calibration of the equipment was verified before performing any measurement.

After, a Matlab/Simulink model was built, compiled and loaded to the controller board. In this model, each sensor signal was conditioned to present the data in the desired units, as well as the output signals that were generated for the excitation of the piezoceramic. In dSPACE ControlDesk, a Graphic User Interface (GUI) was also designed to show all the desired information and to enable an interface to save the experimental data. A picture of the Simulink model and the ControlDesk GUI are presented in Figure 25 and Figure 26.
Chapter 4: Experimental Set-up

Figure 25 - Simulink model for experimental signal generator and data acquisition

Figure 26 - ControlDesk GUI for data storage
4.4. Measurements of a Piezoceramic Actuator (MIDE)

For the purpose of this investigation, a PA manufactured by MIDE [61] model QP20W was selected. It is a Bimorph PZT which was selected to be used under the double-layer extension mode (see Figure 5), providing a large extension/compression force. Table 3 shows the manufacturer specifications:

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size [in]</td>
<td>2.00 x 1.50 x 0.03</td>
</tr>
<tr>
<td>Weight [oz]</td>
<td>0.28</td>
</tr>
<tr>
<td>Capacitance [F]</td>
<td>0.2 x 10^-6</td>
</tr>
<tr>
<td>Voltage range [V]</td>
<td>± 200</td>
</tr>
<tr>
<td>d_{31} [mV]</td>
<td>-179 x 10^-12</td>
</tr>
<tr>
<td>d_{33} [mV]</td>
<td>350 x 10^-12</td>
</tr>
<tr>
<td>g_{21} [V/m/N]</td>
<td>24.2 x 10^-2</td>
</tr>
<tr>
<td>g_{31} [V/m/N]</td>
<td>-11.0 x 10^-3</td>
</tr>
<tr>
<td>Y_{11} [GPa]</td>
<td>69</td>
</tr>
<tr>
<td>Y_{33} [GPa]</td>
<td>55</td>
</tr>
<tr>
<td>K_T [ ]</td>
<td>1800</td>
</tr>
<tr>
<td>Density [kg/m^3]</td>
<td>7700</td>
</tr>
</tbody>
</table>

Table 3 - MIDE QP20W specifications

Since the resonant method applies basically for piezoelectric crystals with different shapes, it is not appropriate for our purposes. Instead, quasi-static and dynamic measurements are here considered for the characterization of the actuator under research.

4.4.1. Measurement Procedure and Obtained Results

The characterization measurements are focused on the different parameters that are involved in the configuration of actuation along axis-1 of the PA. In this section, a description of all performed measurements is presented, as well as obtained results.
4.4.1.1. Measuring Hysteresis

- Dynamic measurements are performed at different frequencies.

- A zero-centred, variable periodic electric field is applied as excitation to its maximum amplitude allowed (± 200V). Positive excitations (non zero-centred) are also applied.

- Different excitation signals are used: sine, triangular, rectangular.

- The piezoceramic lies under no-stress in all axes, allowing a free-strain condition.

- Electric charge and strain along axis-1 are measured. The electric charge can be obtained by means of the electric current, as charge is the integral value of electric current: \( Q = \int I \, dt \)

- For each excitation type, the obtained plots are: Input Voltage vs. Strain, Input Voltage vs. Electric Charge.

Obtained results:

- A hysteretic behaviour of voltage vs. strain is obtained as shown in Figure 28. Hysteresis can be measured as the percentage of peak-to-peak strain at a zero voltage divided by maximum the peak-to-peak strain. The resulting voltage vs. strain hysteresis is of 22%.

- A hysteretic behaviour of 28% of voltage vs. electric charge is obtained as shown in Figure 28 (initial residual strain and charge are present in hysteresis plots due to previous performed measurements; nevertheless, it does not affect the hysteresis loop).

- Residual strain and charge due to hysteresis can be observed in Figure 29 applying a positive voltage input. This is the strain and charge difference that remains when the voltage returns to zero, resulting in 23\(\mu\)e and 7\(\mu\)C.
4.4.1.2. Measuring Frequency Response

- Dynamic measurements are performed, using a frequency sweep input.

- A sinusoidal voltage with amplitudes lower than the maximum allowed is applied, preventing the voltage amplifier to overload due to the increasing power requirements at higher frequencies as shown in equation (42).

- The piezoceramic lies under no-stress in all axes, allowing a free-strain condition.

- Electric charge and strain along axis-1 are measured. Amplitude and phase variations due to the input frequency are then analyzed.

Obtained results:

- After data processing, it was concluded that the actuator presents a fundamental frequency around 500 Hz. Figure 30 shows a Frequency Response Function (FRF) plot of the piezoceramic.
• The actuator approaches a second order lineal model below its resonance frequency, as presented in Figure 30, where the black points represent measurements taken from the actuator, and the blue line represents a second order linear model curve fitted to match the experimental results.

• Fitting of the model provided information for the mechanical coefficients of the electro-mechanical model. First, the actuator’s mass \( (m) \) was experimentally measured, and stiffness \( (k) \) and damping \( (b) \) were calculated to best fit the experimental results, representing the blue line on Figure 30.

• A relationship between input charge and output strain can be obtained from the FRF plot at frequencies below resonance, resulting in a value of 2.7. This represents the inverse linear value of the electro-mechanical couple \( n_{em} \).

• Electric charge vs. strain plots at different frequencies show a phase lag when operating at higher frequencies due to the frequency response (Figure 31).

![Figure 30 - Frequency response of strain/charge](image)
4.4.1.3. Measuring Piezoelectric Strain Coefficients \( (d_{mi}) \)

- Quasi-static measurements are performed.

- An electric field is used as the excitation in the polarization axis \( (3) \), with amplitude of the maximum recommended for the actuator \( (\pm 200V) \).

- The piezoceramic lies under no-stress in all axes, allowing a free-strain condition, thus under the assumption of: \( S_i = d_{3i}E_3 \)

- Strain \( S_i \) is then measured, and the strain coefficient \( (d_{3i}) \) is calculated depending on the input electric field.

**Obtained results:**

- \( d_{31} \) and \( d_{32} \) were obtained as shown in Figure 32 and Figure 33. They present a slight nonlinear behaviour when the electric field reaches the actuator limit, especially in the negative region.

- \( d_{31} \neq d_{32} \), differing from the constitutive equations by IEEE [19]. This is in agreement with some references indicating that the material has an anisotropic configuration at a micro scale [25].

- The coefficients present an average variation from the ones established from the manufacturer. The manufacturer established a value of \( d_{31} = d_{32} = 179 [\text{pm/V}] \), and the obtained data show average values of \( d_{31} = 224 [\text{pm/V}] \) and \( d_{32} = 255 [\text{pm/V}] \).
This represents a variation of the maximum output strain from $\pm 141\mu e$ to $\pm 175\mu e$, and an error of 24%.

- Causes of difference in the coefficient values may be accounted to the manufacturing procedure, or to the experimental measuring conditions (e.g. temperature characteristic) that might affect the PA. A research document [12] from the same manufacturer also demonstrates how the real values differ from those specified by the manufacturer [61], analyzing two products and obtaining a maximum error of 37%.

![Figure 32 - Non-linear input voltage vs. strain behaviour](image)

![Figure 33 - $d_{31}$ and $d_{32}$ coefficients as functions of input voltage](image)
4.4.1.4. Measuring Compliance Coefficients \( (s^E_{ij}) \)

- Quasi-static measurements are performed.

- A constant DC voltage is applied and maintained during each measurement. Different voltage amplitudes are selected.

- A variable external force is used as input to excite axis-1 to a maximum allowed by the force sensor. It is noticed that this experiment will only include the compression property, since tensioning the actuator relies in a more complex test bench.

- The piezoceramic lies under no-stress in axes 2 and 3, thus validating the next expressions: \( T_2, T_3 = 0 \) \( S_1 = s_{11}T_1 + d_{31}E_3 \)

- Strain is then measured, and the compliance coefficient \( (s^E_{11}) \) is calculated for several voltages, thus, the blocked force for each voltage level can be extrapolated.

Obtained results:

- \( s^E_{11} \) was obtained as shown in Figure 34. It showed an approximate linear tendency inside the range that it was measured. It showed a variation of manufacturer’s data which stated to be \( s^E_{11} = \frac{1}{Y^E_{11}} = 14.4 \times 10^{-12}[\frac{m^2}{N}] \) and the experiments showed a value of \( s^E_{11} = 18.8 \times 10^{-12}[\frac{m^2}{N}] \).

- Encapsulation of the piezoceramic material into a composite material to form the actuator may affect the variation of this parameter, as this coefficient is defined for the piezoelectric material and the measurements were made to the complete actuator.

- A performance plot of the PA which demonstrates equation (19) was generated. In this plot, the relation between force, strain and voltage can be visualized (Figure 35). Performance at 0 V according to the manufacturer’s specifications is also plotted.

- An approximate blocked force of \( \pm 77N \) can be obtained with a \( \pm 200V \) input voltage, as shown in Figure 35.
Figure 34 - Elastic compliance coefficient $s_{E11}$

Figure 35 - Force vs. strain performance
Chapter 5

Piezoceramic Actuator Model Characterization and Validation

After performing the characterization procedures, a complete model of the PA, for simulation and control purposes, was developed in Matlab/Simulink according to the scheme in Figure 36. This model accounts for voltage-to-strain nonlinearities, hysteresis and electro-mechanical dynamic effects. The hysteresis is based on a Maxwell slip model, where the non-linear voltage-to-charge properties of the piezoceramic are presented as a series of voltage-limited capacitors. After developing the basic model, an inverse block model (Figure 50) which predicts the complex behaviour of the actuator, including the studied nonlinearities, was implemented to interact with a physical system. After processing a force control signal established by the user, the model determines the output voltage to be delivered to the actuator.

For the hysteresis characterization of the PA, a rising voltage vs. charge curve (Figure 29) was used. The capacitance and voltage parameters obtained are presented in Table 4. The linear fit of the curve was done dividing it into 11 elements.
The dynamic behaviour of the PA can be modelled by a second order model \[21, 23, 24, 46, 56, 58\] as observed in the characterization experiments. First the mass \(m\) was measured and the stiffness \(k\) and damping \(b\) were numerically determined by a frequency response curve approximation. \(C\) was determined by model fitting using the manufacturer’s value as a reference.

Nonlinearities were found in the piezoelectric strain coefficient \(d_{31}\) as shown in Figure 33. As explained in equations (40) and (41), this coefficient affects directly the electro-mechanical couple \(n_{em}\), therefore, it can be modelled as a polynomial function dependent on the input voltage.

It was also expected to obtain a non-linear compliance coefficient \(s_{11}\) that would also be non-linear, but experiments yielded linear coefficients only. This could be due to the narrow operational strain range.

The resulting parameters used to build the model and simulations are:

### Table 4 - Parameters used for the hysteretic Maxwell model

<table>
<thead>
<tr>
<th>Element</th>
<th>Capacitance (C_i) [(\mu)C/V]</th>
<th>Break voltage (v_i) [V]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.44</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>2.96</td>
<td>1.05</td>
</tr>
<tr>
<td>3</td>
<td>2.10</td>
<td>3.96</td>
</tr>
<tr>
<td>4</td>
<td>2.18</td>
<td>7.64</td>
</tr>
<tr>
<td>5</td>
<td>2.84</td>
<td>9.16</td>
</tr>
<tr>
<td>6</td>
<td>3.28</td>
<td>11.09</td>
</tr>
<tr>
<td>7</td>
<td>3.80</td>
<td>12.31</td>
</tr>
<tr>
<td>8</td>
<td>3.61</td>
<td>15.55</td>
</tr>
<tr>
<td>9</td>
<td>2.17</td>
<td>28.80</td>
</tr>
<tr>
<td>10</td>
<td>3.61</td>
<td>17.86</td>
</tr>
<tr>
<td>11</td>
<td>0.77</td>
<td>87.36</td>
</tr>
</tbody>
</table>

### Table 5 - Parameters for the MIDE QP20W piezoceramic actuator model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>(m)</td>
<td>0.001995</td>
<td>kg</td>
</tr>
<tr>
<td>Stiffness</td>
<td>(k)</td>
<td>(1.8 \times 10^4)</td>
<td>N/m</td>
</tr>
<tr>
<td>Damping</td>
<td>(b)</td>
<td>2.4</td>
<td>N-s/m</td>
</tr>
<tr>
<td>Linear Capacitance</td>
<td>(C)</td>
<td>(0.2 \times 10^{-7})</td>
<td>F</td>
</tr>
<tr>
<td>Electro-mechanical couple</td>
<td>(n_{em})</td>
<td>(f (V_m))</td>
<td>C/m</td>
</tr>
</tbody>
</table>

\(n_{em} = - 3.757 \times 10^{-16} V_m^6 - 8.363 \times 10^{-14} V_m^5 + 4.072 \times 10^{-11} V_m^4 + 7.155 \times 10^{-9} V_m^3 - 8.042 \times 10^{-7} V_m^2 - 2.096 \times 10^{-4} V_m + 3.574 \times 10^{-3}\)
5.1. Model in Matlab/Simulink

The resulting Simulink model of the PA is presented from Figure 37 to Figure 40. The hysteretic Maxwell slip-model was programmed inside the “Hysteresis model” block and described in Appendix B. This complete model is based on equation (1) or in Figure 14b, and the parameters used are those presented in Table 5.

Figure 37 presents the complete block model. It has a voltage and an external force as inputs, and displacement as output from the model. Each one of the main blocks is presented in the next figures. Figure 38 represents the electrical domain of the actuator and Figure 39 the Maxwell hysteresis model. Figure 40 presents the mechanical domain of the PA, which is represented by a linear second order dynamic model.

Figure 37 - Hysteretic dynamic non-linear block model

Figure 38 - Non-linear electrical domain model
5.2. Model Validation

Results from the model here presented were compared with experimental data as shown in Figure 41 to Figure 47. Each Figure presents three plots: the input voltage to the actuator, the output strain from experiments and from the model, and the full scale range (FSR) error percentage between the compared strains. This error is obtained from the difference of the two signals and divided by the FSR strain (maximum value that can be obtained from the PA).
In Figure 41, the actuator was excited at 1 Hz with a zero-centred sinusoidal voltage of ±200 V, with the objective of obtaining the maximum operational loop of the PA. Hysteresis between the input voltage and the resulting strain is observed. This hysteresis mayor loop has an average error of 1.02%, being the maximum of 5.69% in the initial raising curve. Some experimental plots present an initial non-zero strain (positive or negative), possibly due to a residual charge or strain on the material after measurements. However, errors in this initial curve do not affect the remaining cycles.

A linearly decaying sinusoid voltage of 1 Hz was used to obtain the plot in Figure 42. This type of input is selected to visualize the hysteretic internal loops. The resulting plot presents an average error of 1.46%. The largest differences are seen on the zoomed lower plots of the figure, with a maximum error of 6.22%. This is possibly due to an electric inertia not captured by the model, which attenuates drastic voltage changes, allowing edged peaks. On the middle plot, an uncommon behaviour is observed on the experimental measurements in the range of 80-100 V. Strain trajectory seems to change drastically inside this range. This behaviour was also present in other performed measurements (see Figure 43). On this plot, an average error of 1.65% and a maximum error of 4.69% were obtained.

The hysteresis relation of input voltage and charge is shown in Figure 44. The voltage input was a sinusoidal voltage of +200 V at 10 Hz. Notice that the initial curve and the maximum charge value of the experimental and model are in very good agreement, although the residual charge when it reaches a zero voltage differs, as well as in the second upward trajectory. This plot presents a higher average error of 2.91%. Even though a larger error is obtained in the voltage-charge relation, the voltage-strain relation, which is of more physical significance, shows a better closer agreement.

From Figure 45 to Figure 47, different input signals were tested and plotted along time. During experimentation, several signals were tested: sinusoidal, triangle, rectangular, step, linearly decaying sinusoidal and triangular; and also at different amplitudes and frequencies. These plots demonstrate that the proposed model is able to predict most of the significant features found in the experimental results. As observed, the largest error (10%) is present at rectangular plots with higher amplitude difference; the reason for this might be the obliged quicker changes, resulting in a higher difference error. Measured
average and maximum FRS error percentage, hysteresis percentage and residual values from the validation plots are presented in Table 6.

Figure 41 - Voltage vs. strain hysteresis for a zero-centred sinusoidal

Figure 42 - Linearly decaying 1 Hz zero-centred sinusoidal voltage input
Chapter 5: Piezoceramic Actuator Model Characterization and Validation

Figure 43 - Voltage vs. strain hysteresis for a positive sinusoidal input

Figure 44 - Voltage vs. charge hysteresis for a positive sinusoidal input

Figure 45 - Strain due to a 10Hz positive triangular input
Regarding the error percentages here obtained, they are put in context to those reported by other researches: An adaptation of the Preisach model was considered in [42] to describe the nonlinear hysteresis behaviour of piezoceramic actuators. The obtained model reproduced the hysteresis loop to within 3% over the entire working range of 0-
15um. A Preisach and a P-I model were compared in [41], and obtained average displacement errors of 0.86 % and 10.26% respectively. A P-I inverse model with different controllers was studied in [52], and obtained a maximum tracking error of 13.17% using a PID controller. Using a new proposed hysteresis model, [58] obtained a maximum tracking error of 10% FSR under open-loop operation. In view of this, it can be stated that the model here introduced achieves a good balance of simplicity and accuracy.

5.3. Alternate model

Based on the same modelling equations (33) to (39), and having previously validated the model with experimental data, a different model based on equation (5) and Figure 14a was also modelled in Simulink. This model differs from the presented in Figure 37 to Figure 40 in that it leaves the input voltage and the actuator strain as independent variables, and the generated force as the output value, consequently, a model based on output strain or output force can be used for control purposes.

The modelling of the hysteresis and the electro-mechanical couple remains the same as in the previous model. The left side of the model refers to the electrical domain, while the mechanical domain lies in the right side.

5.4. Inverse Piezoceramic model

Inverse models have been proposed for different actuator models [21, 44, 52, 53], which help reduce the non-linear behaviour mainly due to hysteresis. In many cases, the inverse
model has to be computed or analytically modified, also based on a reference model. The inverse model here proposed is completely based on the same equations ((33) to (39)) of the PA model, with variations only on the definition of Simulink block model.

After completing the models that represent the PA, an inverse control model was developed. For an open-loop operation, this model linearizes the PA behaviour. Figure 49 shows a representation of this concept. This model is able to compensate the hysteresis, nonlinearities and dynamics of the physical actuator, in order to meet a displacement (or force) target. For this inverse model, a feed-backward signal with the instant strain of the actuator and a control signal of the desired output force to be delivered need to be provided (or vice versa), resulting in the calculation of the input voltage needed to be provided to the actuator to execute the target control strain (or force).

![Figure 49 - Inverse model scheme for behaviour linearization](image)

This model is based on the same equations ((33) to (39)) as the actuator models previously presented. In this case, voltage is the output signal, which represents the input voltage of the actuator to accomplish the force and strain signals. The mechanical domain can be seen on the left side of the model (Figure 50), and the electrical domain on the right side. This inverse model used in conjunction to the models presented in Figure 37 and Figure 48 represents the linearized feed-forward output model.

![Figure 50 - Inverse model of the piezoceramic actuator](image)
Chapter 6

Simulation of the Mechanical System-Actuator Interaction

6.1. Interaction with Mechanical System

One of the main application fields of smart materials is known as “smart structures”. In this, a previously characterized smart material (i.e. a PA) is coupled to a traditional structure, aiming at exerting continuously variable control over a determined feature of the structure’s response. Due to the coupling, the smart material exhibits a mechanical interaction with the host structure, exerting and receiving forces to and from the latter. This complex scenario is ideal to test the validity of mathematical models of smart materials, since main effects (hysteresis, non-linearity, dynamics, etc.) are likely to play a significant role.

In this research, the interaction between a PA and a simple mechanical system is simulated. Consider a pair of piezoelectric patch actuators surface-bonded to a fixed-free (cantilever) beam, which is, in turn, under transversal displacement due to an external force applied at the tip. The mechanical interaction is caused when an external voltage is applied to the piezoceramic, which, in agreement with its constitutive equation, would yield a longitudinal strain. Given that the piezoceramic is bonded to the beam’s surface, thus it barely deforms, rather applying axial forces along the beam. This new scenario modifies the transversal displacement of the beam, which in turns exerts new forces back to the piezo, and so on. Figure 51 shows this interaction:
A cantilever beam can be represented by a second order linear transfer function (TF) (equation (43)) with its fundamental natural frequency defined by equation (44), where $m_{eq}, c, k_{eq}$ are the equivalent mass, damping and stiffness of the model, respectively.

\[
TF = \frac{1}{m_{eq}s^2 + cs + k_{eq}} \tag{43}
\]

\[
f_n = \frac{1}{2\pi} \sqrt{\frac{k_{eq}}{m_{eq}}} \tag{44}
\]

The interaction of the actuator to a cantilever beam can be represented as an axial load which is then transmitted as a pair of acting moments $M$ in the limits of the actuator (equation (45)), as shown in Figure 52:
Chapter 6: Simulation of the Mechanical System-Actuator Interaction

Figure 52 - Mechanical system interaction

\[ M = \frac{F_a (t_a + t_b)}{2} \]  

\( F_a \) represents the output force of the PA, \( F_t \) the force applied at the tip of the beam, \( t_a, t_b, l_a \) and \( l_b \) the actuator and beam thickness and length, respectively. \( M \) is the generated moment due to the actuator force, \( x_a \) the starting position of the actuator from the fixed side of the beam, \( e_a \) the actuator’s average strain along the x axis, and \( z_t \) the tip displacement along the z axis. For the simulation of this mechanical interaction, a PA and a beam with dimensions and properties shown in Table 3 and Table 7 were selected.

<table>
<thead>
<tr>
<th>Material</th>
<th>Aluminium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>2768 [kg/m³]</td>
</tr>
<tr>
<td>Young modulus</td>
<td>7.31 x10¹⁰ [N/m²]</td>
</tr>
<tr>
<td>Length</td>
<td>0.22 [m]</td>
</tr>
<tr>
<td>Width</td>
<td>0.032 [m]</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.001 [m]</td>
</tr>
</tbody>
</table>

Table 7 - Beam properties

A finite element model (FEM) was built in ANSYS (see Figure 53), to simulate the beam-piezoceramic interaction (for more details about the FE modelling procedure, refer to Appendix D). The purpose of this model was to obtain the relationship of the actuator force \((F_a)\) with the force applied at the tip of the beam \((F_t)\) with the assumption of a linear behaviour; and the relation of the tip displacement \((z_t)\) with the average strain in the PA \((e_a)\). The obtained values are presented in equation (46) and (47). These values represent the mechanical interaction of the PA and the cantilever beam, which is then modelled in Simulink for simulations.

\[ \frac{F_a}{F_t} = -2634.7 \text{ [N/N]} \]  

(46)
Using properties of Table 7, a transfer function for the beam, in Laplace domain, with a tip force input and tip displacement output is obtained as presented in equation (48). Values of mass and stiffness were mathematically obtained using the properties and dimensions of the beam. To obtain the damping and natural frequency, a physical experiment was implemented, based on logarithmic decrement procedures.

\[
\frac{Z_t}{F_t} = \frac{1}{[0.0477s^2+0.044996s+54.9211]}
\] (48)

### 6.2. Open-loop control simulation

A simulation for open-loop control actuator positioning is now analyzed. The objective of this simulation is to demonstrate the difference between linear constitutive equations and the simulated behaviour of PAs including non-linear characteristics. For this example, a signal of input force and input voltage are established. These two input signals enter the piezoceramic model already developed (see Figure 37) which is represented by block "Piezoceramic model A" in Figure 54. The output of the model is the strain of the PA,
which is compared to a reference strain signal generated by a linear constitutive equation (see equation (1)).

Figure 55 presents the input, output and error signals. A 0-100V, 5Hz sinusoidal input voltage and a ±20N, 1Hz sinusoidal input force are shown on the left plot. The plot on the right side shows the strain reference signal based on linear constitutive equations (continuous line), superimposed with the output strain from the PA model previously developed (dashed line). Both signals present a difference error attributed to the nonlinearities, hysteresis and dynamic behaviour present on real actuators. The FSR error percentage presents an average of 4.83% and a maximum value of 11.27%.

![Figure 54 - Open-loop model comparison](image)

![Figure 55 - Open loop model comparison signals](image)
Now, a simulation is presented to validate the functionality of the proposed inverse model of the PA. The inverse model (see Figure 50) is now inserted to the previous diagram for an open-loop control scheme. The force signal enters both block models, and the same reference signal for the desired strain inputs the inverse model. This model will calculate the input voltage needed for the PA to achieve the desired strain, therefore, linearizing the actuator.

It can be seen in Figure 57 that the reference and output strains follow almost the same trajectory, as desired, having an average error of 0.06% and a maximum value of 0.14%. This error has been minimized from a value of 11.27% of the previous simulation. The input voltage signal must be compared to the one in Figure 55, and a difference is observed. Its amplitude varies with time, compensating the nonlinear behaviour present in the actuator.
6.3. Closed-loop control simulation

In this section, two closed-loop control simulations are presented. The purpose of these simulations is to demonstrate the interaction of a PA with a mechanical system, and at the same time, to validate the functionality of the alternate proposed model, which exerts an output force instead of a displacement. The mechanical system presented in Figure 52 is here considered. The transfer function of the beam and the mechanical interaction with the PA previously defined are used as shown in Figure 58. This interaction transforms the tip displacement into an average strain in the PA, and the actuator force into an acting force on the tip of the beam, assuming linear relationships. For this simulation, the alternate piezoceramic model (Figure 48) is used. A force control is desired instead of a strain or position control. The same inverse model can be used for both piezoceramic models.

On the simulation, first a direct sinusoidal force excites the tip of the beam at its natural frequency, causing the beam to resonate. After four seconds, the force is removed, and the actuator begins to attenuate the vibrations using a Proportional Integral Derivative (PID) controller. Figure 59a, shows the input force as a continuous line, and the dashed line represents the controlled force of the actuator. It can be seen on Figure 59c how the controlled output stabilizes to zero after less than 2 seconds, while uncontrolled beam continues to vibrate well after this time.
Another type of closed-loop control is considered in Figure 60. The mechanical system is the same as presented in Figure 58, the difference remains on the control loop. Now, a specific tip displacement is desired, represented by a reference signal block. A typical close-loop scheme is used, with a PID controller.

Figure 61a shows the control force signal (dashed line) and the generated control voltage signal (continuous line) after the inverse model. Shown in Figure 61b, as the system is a second order under-damped model, it will tend to oscillate within stepped inputs, but the actuator counteracts and reduces the oscillating time and leaves the tip displacement at the desired reference position.
6.4. Summary

A basic mechanical interaction between a PA and a cantilever beam is here presented. The use of the inverse piezoceramic model is used for an open-loop control to demonstrate the importance of this model for strain or position control. It proves to be a good complement the actuator model, compensating the hysteresis, nonlinear coefficients and dynamic behaviour, resulting in a linear input/output relation. The implementation in simulations of an inverse model represented a decreasing FSR error percentage of the PA in open-loop operation, from a maximum value of 11.27% to 0.14%.

Closed-loop control schemes with basic control theory also show a good response for the proposed PA models. Vibration attenuation and reference tracking control using PID controllers, and interacting with a mechanical system, can be achieved using the proposed models.
Chapter 7

Conclusions and Future Work

7.1. Conclusions

- Developments in this thesis allow a deeper insight on the behaviour of piezoelectric materials, mainly piezoceramic actuators. Many types of nonlinearities (hysteresis, material nonlinearity, piezoelectric nonlinearity, frequency response, creep, aging, thermal behaviour, etc.) can be found in performing piezoelectrics, but the more representative were considered in this research: material nonlinearity, piezoelectric nonlinearity, hysteresis, and frequency response.

- Linear constitutive equations of piezoelectrics are the foundation of the electro-mechanical model here presented. This model represents a more realistic model of piezoelectrics, splitting it into electric and mechanical domain. It also allows the straightforward inclusion of the hysteresis model as well as the dynamic model behaviour.

- Hysteresis in piezoelectric materials was shown to be significant. Thus, it can induce great variability in the expected behaviour if not compensated.

- The Maxwell slip model here characterized is widely used in the definition of hysteresis modelling in different cases, including piezoelectric materials. Although not exact, this model can approximate the hysteretic behaviour of a typical PA, with the advantage of requiring a single experiment for characterization plus a little of data extrapolation.

- It was shown that piezoelectric strain coefficients behave non-linearly with the applied voltage. The non-linearity is stronger when approaching higher voltage limits, especially for negative voltages. It was also demonstrated that the obtained properties did not coincide with the data provided from the manufacturer, which further gives more importance to the characterization when using new actuators.

- In spite of literature claims stating the opposite, the elastic compliance coefficients here analyzed behaved entirely linear within the measured range (compressive loads only).
• Frequency response measurements supported the assumption that a PA can be represented by a second order lineal model below its fundamental frequency.

• The experimental procedures here presented, based on quasi-static and dynamic measurements, allow a complete full characterization of the actuator. These procedures are applicable to arbitrary arrangements of piezoelectrics (i.e. beyond the double layer extension mode type).

• Although the experimental set-up for a full and reliable characterization requires specialized equipment, the procedures are straightforward in practice.

• Based on the model validation, a maximum average FSR error of 2.95% was obtained compared to experimental results. Compared to other research [41, 42, 52, 58], the GMS hysteresis model results to be more accurate than the P-I model; on the other hand, the Preisach model represents a more accurate model, but it has a time consuming parameter estimation procedure.

• The developed block model in Matlab/Simulink was validated by comparing numerical and experimental results, accounting for hysteresis, nonlinearities and dynamic behaviour. In comparison with the linear constitutive equations presented by IEEE [19], the developed model is much more accurate for predicting the overall structural behaviour of the actuator.

• The formulated mathematical model proved of great versatility, allowing building an alternate model and an inverse model of the PA. This inverse model, together with the actuator model, counteracts the nonlinearities on the piezoelectric, linearizing the output / input in an open-loop control scheme.

• Basic close-loop control schemes were used to validate the proposed model with a mechanical system interaction. They proved to work for vibrations attenuation and force target control.

• The implementation in simulations of an inverse model represented a decreasing FSR error percentage of the PA in open-loop operation, from a maximum value of 11.27% to 0.14%.
7.2. Future Work

- The scope of this research was limited to the experimental validation of piezoelectrics models, and the applicability of the developed models within simulated control schemes. Therefore, physical validation of the latter should be performed before drawing final conclusions.

- A representative electric non-linear behaviour was considered while modelling a PA. In spite of literature claims, nonlinearities in the elastic compliance coefficients could not be proven, at least within the specified measurement range. However, this and other non-linear features, such as creep, temperature behaviour, and aging -among others-, could be included to develop a more accurate model.

- The characterization and modelling procedure should be tested for other types of PA modes, such as stacks, transverse or longitudinal motor types.

- An enhanced test bench that could induce tension loads in the actuator should be considered, thus new experimental measurements ought to be taken.

- PAs are known to have an increasing interest among researchers and engineers, therefore, new applications based on this model could be developed.
References


2. Linear Theory of Piezoelectricity

2.1 General

In linear piezoelectricity the equations of linear elasticity are coupled to the charge equation of electrostatics by means of the piezoelectric constants. However, the electric variables are not purely static, but only quasistatic, because of the coupling to the dynamic mechanical equations. Thus, in order to provide an appropriate theoretical basis for the material covered in this standard, the relevant mechanical and electrical field variables will be briefly defined and the pertinent mechanical and electrical equations presented in this section.

2.2 Mechanical Considerations

The Cartesian components of the infinitesimal mechanical displacement of a material point are denoted by $u_i$.

NOTE — Cartesian tensor notation is used throughout this standard. See Jeffreys [B1]. For a more complete discussion of mechanical displacement, see Tersten ([B2], Chapter 3, Section 1).

The symmetric portion of the spatial gradient of the mechanical displacement determines the strain tensor $S_{ij}$.

NOTE — A comma followed by an index denotes partial differentiation with respect to a space coordinate.
Thus,

\[ S_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \]  \hspace{1cm} (1)

where

\[ u_{ij} = \partial u_{i}/\partial x_{j} \]

The antisymmetric portion of the mechanical displacement gradient determines the infinitesimal local rigid rotation ([B2], Chapter 3, Section 2.), which is allowed to take place without constraint in the continuum, and is of no consequence in this standard. The velocity of a point of the continuum is given by

\[ v_{i} = u_{i} = \partial u_{i}/\partial \tau \]  \hspace{1cm} (2)

where \( \tau \) denotes the time. The mass per unit volume is denoted by \( \rho \), which will be a constant for any material throughout this standard.

The mechanical interaction between two portions of the continuum, separated by an arbitrary surface \( S \), is assumed to be given by the traction vector, which is defined as the force per unit area \( T_{j} \) acting across a surface at a point and dependent on the orientation of the surface at the point.

NOTE — For a more complete discussion of traction, see ([B2], Chapter 2, Section 1.).

In fact the existence of the traction vector and the integral form of the equations of the balance of linear momentum determine ([B2], Chapter 2, Section 2.) the existence of the stress tensor \( T_{ij} \), which is related to the traction vector \( T_{j} \) by the relation

\[ T_{j} = n_{j} T_{ij} \]  \hspace{1cm} (3)

where \( n_{j} \) denotes the components of the outwardly directed unit normal to the surface across which the traction vector acts. Clearly, \( T_{ij} \) is a second-rank tensor.

NOTE — The summation convention for repeated tensor indices is employed throughout. See [B1], Chapter 1.

From Eq 3 and the integral forms of the equations of the balance of linear momentum result the stress equations of motion:

\[ T_{i,j} = \rho u_{j} \]  \hspace{1cm} (4)

where, from the conservation of angular momentum, the stress tensor \( T_{ij} \) is symmetric.

In linear theory the components of the vectorial flux of mechanical energy across a surface are given by \( -T_{ij} \).

### 2.3 Electrical Considerations

In piezoelectric theory the full electromagnetic equations are not usually needed. The quasiaelectrostatic approximation is adequate because the phase velocities of acoustic waves are approximately five orders of magnitude less than the velocities of electromagnetic waves.

NOTE — For more detail concerning the nature and limitations of the approximation, see ([B2], Chapter 4, Section 4.).
Under these circumstances magnetic effects can be shown to be negligible compared to electrical effects. In electrical theory the Cartesian components of the electric field intensity and electric displacement are denoted, respectively, by $E_i$ and $D_i$. In MKS units these two vectors are related to each other by

$$D_i = \varepsilon_0 E_i + P_i$$  \hspace{1cm} (5)

where $P_i$ denotes the components of the polarization vector, and the permittivity of free space $\varepsilon_0$ is given by

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$  \hspace{1cm} (6)

The electric field vector $E_i$ is derivable from a scalar electric potential $\phi$:

$$E_i = -\nabla \phi$$  \hspace{1cm} (7)

The electric displacement vector $D_i$ satisfies the electrostatic equation for an insulator,

$$D_{i;i} = 0$$  \hspace{1cm} (8)

It should be noted that although the electric field equations appear to be static, they are time dependent because they are coupled to the dynamic mechanical equations presented in 2.2. The time-dependent vector flux of electrical energy across a surface is given by $(+ \phi_i)$, which is the degenerate form taken by the Poynting vector in the quasistatic electric approximation.

NOTE — For details concerning the derivation of the degenerate form of the Poynting vector for the quasistatic electric field, see [B2], Chapter 4, Sections 3. and 4.).

2.4 Linear Piezoelectricity

The conservation of energy ([B2], Chapter 5, Sections 1–3.) for the linear piezoelectric continuum results in the first law of thermodynamics:

$$U = T_{ij} S_{ij} + E_i D_i$$  \hspace{1cm} (9)

where $U$ is the stored energy density for the piezoelectric continuum. The electric enthalpy [B3] density $H$ is defined by

$$H = U - E_i D_i$$  \hspace{1cm} (10)

and from Eqs 9 and 10 there results

$$H = T_{ij} S_{ij} - D_i E_i$$  \hspace{1cm} (11)

Eq 11 implies

$$H = H(S_{ik} E_k)$$  \hspace{1cm} (12)
and from Eqs 11 and 12 there result

\[ T_{ij} = \partial H / \partial S_{ij} \]  
(13)

\[ D_i = -\partial H / \partial E_i \]  
(14)

where it should be noted that

\[ \partial S_{ij} / \partial S_{jk} = 0, \quad i \neq j \]  
(15)

in taking the derivatives called for in Eq 13.

In linear piezoelectric theory the form taken by \( H \) is

\[ H = \frac{1}{2} c_{ijkl}^{kk} S_{ij} S_{kl} - c_{ijkl}^{kl} E_{k} S_{ij} - \frac{1}{2} c_{ij}^{kl} E_{i} E_{j} \]  
(16)

where \( c_{ijkl}^{kk} \), \( c_{ijkl}^{kl} \), and \( c_{ij}^{kl} \) are the elastic, piezoelectric, and dielectric constants, respectively. In general there are 21 independent elastic constants, 18 independent piezoelectric constants, and 6 independent dielectric constants. From Eqs 13, 14, and 16 with Eq 15 there result the piezoelectric constitutive equations:

\[ T_{ij} = c_{ijkl}^{kl} S_{ij} S_{kl} - c_{ijkl}^{kl} E_{k} \]  
(17)

\[ D_i = c_{ijkl}^{kl} S_{ij} + c_{ij}^{kl} E_{k} \]  
(18)

Note that the substitution of Eqs 10 and 18 into Eq 16 yields

\[ U = \frac{1}{2} c_{ijkl}^{kk} S_{ij} S_{kl} + \frac{1}{2} c_{ij}^{kl} E_{i} E_{j} \]  
(19)

and there is no piezoelectric interaction term in the positive definite stored energy function \( U \). Since the \( c_{ij}^{kl} \) do not appear in Eq 19, the positive definiteness of \( U \) places restrictions on the \( c_{ijkl}^{kl} \) and the \( c_{ij}^{kl} \), but not on the \( c_{ijkl}^{kk} \). Note further that the substitution of Eqs 1 and 9 into Eqs 17 and 18, and then Eqs 17 and 18 into Eqs 4 and 8 yields the four differential equations

\[ c_{ijkl}^{kl} \partial_{k} u_{i j} + c_{ijkl}^{kl} \varphi_{k i} = \rho \varphi_{j} \]  
(20)

\[ c_{ijkl}^{kl} \partial_{k} \varphi_{i j} - c_{ij}^{kl} u_{k i j} = 0 \]  
(21)

in the four dependent variables \( u_{ij} \) and \( \varphi \). Eqs 20 and 21 are the three-dimensional differential equations for the linear piezoelectric continuum.

No notational distinction between the isothermal and adiabatic material constants is made in this standard. The scalar symbols \( \theta \) and \( \phi \) are recommended for temperature and entropy to avoid confusion with the tensor symbols \( \partial_{ij} \) and \( T_{ij} \) employed in this standard. Of course, under rapidly varying conditions, the adiabatic values of the constants are understood, and under slowly varying or static conditions, the isothermal values are understood. The form of the constitutive equations given in Eqs 17 and 18 is the only one that is useful for the three-dimensional continuum when
no boundaries are present, because it is the only form that yields Eqs 20 and 21 from Eqs 4 and 8. There are, however, other forms of the constitutive equations, and these become useful in certain specific instances when boundaries are present. The alternate forms that appear most frequently in the literature are presented and discussed in 2.6.

In order to write the elastic and piezoelectric tensors in the form of a matrix array, a compressed matrix notation is introduced in place of the tensor notation, which has been used exclusively heretofore. This matrix notation consists of replacing $ij$ or $kl$ by $p$ or $q$, where $i, j, k, l$ take the values 1, 2, 3 and $p, q$ take the values 1, 2, 3, 4, 5, 6 according to Table 2.

The identifications

$$\begin{align*}
\epsilon_{ijkl} &= \epsilon_{ij}^E, \\
\sigma_{ijkl} &= \sigma_{ij}, \\
T_{ij} &= T_p
\end{align*}$$

are made. Then the constitutive Eqs 17 and 18 can be written:

$$\begin{align*}
T_p &= \epsilon_{ij}^E S_{ij} - \sigma_{ij} X_j \\
D_p &= \epsilon_{ij} S_{ij} + \epsilon_{ij}^P E_j
\end{align*}$$

where

$$\begin{align*}
S_{ij} &= S_p & \text{when } i = j, p = 1, 2, 3 \\
2S_{ij} &= S_p & \text{when } i \neq j, p = 4, 5, 6
\end{align*}$$

NOTE — The summation convention for all repeated indices is understood.

<table>
<thead>
<tr>
<th>Table 2 — Matrix Notation</th>
<th>$ij$ or $kl$</th>
<th>$p$ or $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>22</td>
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<td>23 or 32</td>
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<td>31 or 13</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>12 or 21</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

It should be noted that when the compressed matrix notation is used, the transformation properties of the tensors become unclear. Hence, the tensor indices must be employed when coordinate transformations are to be made. It should also be noted that the time-dependent vector flux of piezoelectric energy is the sum of the mechanical and electrical terms mentioned in 2.2 and 2.3, respectively, and is given by

$$-(T_{ij}\dot{n}_j - q_i D_i)$$
2.5 Boundary Conditions

In the presence of boundaries the appropriate boundary conditions must be adjoining the differential Eqs 20 and 21 of the linear piezoelectric continuum. If there is a material surface of discontinuity, then across the surface there are the continuity conditions ([32], Chapter 3, Section 4; Chapter 6, Section 4.)

\[ n_i T_{ij} = n_j T_{ij} \]  \hspace{1cm} (27)

\[ \nu_i^1 = \nu_i^\Pi \]  \hspace{1cm} (28)

\[ n_i D_i^1 = n_i D_i^\Pi \]  \hspace{1cm} (29)

\[ \psi^1 = \psi^\Pi \]  \hspace{1cm} (30)

where I indicates the values of the variables on one side and \( \Pi \) the values of the variables on the other side of the surface of discontinuity, and \( n_i \) denotes the components of the unit normal to the surface. At a traction-free surface, the boundary conditions, Eq 27, become

\[ n_i T_{ij} = 0 \]  \hspace{1cm} (31)

At a displacement-free surface, the boundary conditions, Eq 28, become

\[ \nu_i = 0 \]  \hspace{1cm} (32)

In more general cases different combinations of Eqs 31 and 32 apply. If the appropriate dielectric constant of the material is large compared to the dielectric constant of air (vacuum), the boundary condition, Eq 29, at an air-dielectric interface becomes, approximately,

\[ n_i D_i = 0 \]  \hspace{1cm} (33)

where \( D_i \) is the electric displacement in the material. On a surface with an electrode \( \psi \) must be either specified or some relation between \( \psi \) and \( n_i D_i \) given. If the electrodes are short-circuited and the reference potential is zero,

\[ \psi = 0 \]  \hspace{1cm} (34)

at each electrode. If a pair of electrodes operates into a circuit of admittance \( Y \), the condition is

\[ I = \int_A n_i D_i \, ds = \pm YV \]  \hspace{1cm} (35)

where the \( \pm \) depends on the orientation of the coordinate axes, \( A \) represents the area of the electrode, and the voltage \( V \) is related to the potential difference according to

\[ V = \psi(1) - \psi(2) \]  \hspace{1cm} (36)
2.6 Alternate Forms of Constitutive Equations

For the unbounded piezoelectric medium, the only form of the constitutive equations which is of any value is given in Eqs 17 and 18. Some other forms of the constitutive equations are

\[ S_{ij} = e_{ijk}T_{kl} + \varepsilon_{ij}E_k \]  \hspace{1cm} (37)

\[ D_i = d_{ijk}T_{kl} + \varepsilon_{ijk}E_k \]  \hspace{1cm} (38)

and

\[ S_{ij} = \varepsilon_{ijk}T_{kl} + s_{ijk}D_k \]  \hspace{1cm} (39)

\[ E_i = -s_{ijk}T_{kl} + \varepsilon_{ijk}D_k \]  \hspace{1cm} (40)

and

\[ T_{ij} = e_{ijkl}S_{kl} - h_{ijk}D_k \]  \hspace{1cm} (41)

\[ E_i = -h_{ij}S_{kl} + \varepsilon_{ijk}D_k \]  \hspace{1cm} (42)

These latter forms of the constitutive equations, although exact, are employed in approximations which are valid under certain limiting circumstances. The utility of any one of these three pairs of constitutive equations depends on the fact that certain variables on the right-hand sides are approximately zero under appropriate circumstances. Consequently, the set to use in a given instance depends crucially on the specific geometrical, mechanical, and electrical circumstances. As an example, for low-frequency vibrations of a rod one would use either Eqs 37 and 38 or Eqs 39 and 40, because under these circumstances all stress components vanish, either exactly or approximately, except for the extensional stress along the length of the rod. However, it is not at all clear whether to use the first or the second set unless more specific information concerning the shape of the cross section and placement of electrodes is given. In fact, in a given instance it is quite possible that a different set of constitutive equations somewhere between the two would be useful.

The relations between the coefficients appearing in the four sets of constitutive equations, Eqs 17, 18 and Eqs 37–42, may be written

\[ e_{ij} = \delta_{ij} = \delta_{pq}, \quad e_{ij} = \delta_{pq} \]

\[ \beta_{ij} = \delta_{ij}, \quad \beta_{ij} = \delta_{ij} \]

\[ e_{ij} = e_{ij} + \varepsilon_{pq}h_{pq}, \quad e_{ij} = \varepsilon_{pq} - h_{pq}\delta_{pq} \]

\[ e_{ij} = \varepsilon_{ij} + D_{ij}e_{ij}, \quad \beta_{ij} = \beta_{ij} - s_{ij}\delta_{ij} \]

\[ E_i = d_{ij}e_{ij}, \quad \beta_{ij} = \beta_{ij} - s_{ij}\delta_{ij} \]

\[ S_{ij} = \varepsilon_{ij} + \varepsilon_{ij}h_{ij}, \quad h_{ij} = 3\varepsilon_{ij} \]

using the compressed notation introduced in 2.4 and where \( i,j,k = 1,2,3 \) and \( p,q,r = 1,2,3,4,5,6 \) and \( \delta_{ij} \) is the 3 x 3 unit matrix and \( \delta_{pq} \) is the 6 x 6 unit matrix. As a consequence of Eqs 22 and 25 the following relations hold:
\begin{align}
\sigma_{ij}^p &= \sigma_{jk}^p, \quad i = j \text{ and } k = l, \quad p, q = 1, 2, 3 \\
\sigma_{ij}^p &= 2\delta_{ij}^p, \quad i = j \text{ and } k = l, \quad p = 1, 2, 3, \quad q = 4, 5, 6 \\
\sigma_{ij}^p &= 4\delta_{ijkl}, \quad i = j \text{ and } k = l, \quad p, q = 4, 5, 6
\end{align}

and similar relationships hold for \( \sigma_{kl}^D \):
\begin{align}
\tau_{ij} &= \tau_{kl}, \quad k = l, \quad p = 1, 2, 3 \\
\tau_{ij}^D &= 2\tau_{ijkl}, \quad k = l, \quad q = 4, 5, 6
\end{align}

\text{and similar relationships hold for } \theta_{kl}. \text{ The piezoelectric constants } h_{ij} \text{ are related to the } h_{ij} \text{ in the same way that } c_{ij} \text{ are related to the } c_{ij}, \text{ and the elastic constants } c_{ijkl}^D \text{ are related to the } c_{ijkl} \text{ in the same way that the } c_{ijkl}^D \text{ are related to the } c_{ijkl}.\]

3. Crystallography Applied to Piezoelectric Crystals

3.1 General

The gap between the treatment of piezoelectric solids using the theoretical concepts of continuum mechanics, as presented in Section 2, and the application of the equations of that section to particular piezoelectric materials is spanned by the branch of science called crystallography. Most piezoelectric materials of interest for technological applications are crystalline solids. There can be single crystals, either formed in nature or formed by synthetic processes, or polycrystalline materials like ferroelectric ceramics which can be rendered piezoelectric and given, on a macroscopic scale, a single-crystal symmetry by the process of poling. Since the theoretical principles developed in Section 2 are presented with the generality of tensor formulations, connection of the theory of Section 2, with real piezoelectric materials requires as a first step the definition of crystal axes within the different crystallographic point groups and the association of the crystal axes with the Cartesian coordinate axes used in mathematical analysis. In addition to axis identification and association, the science of crystallography provides a highly developed nomenclature and a wealth of data useful to engineers and scientists working with piezoelectric crystals. Such data are, for example, atomic cell dimensions and angles, interfacial angles, optical properties, and X-ray properties.

3.2 Basic Terminology and the Seven Crystal Systems

The term crystal is applied to a solid in which the atoms are arranged in a single pattern repeated throughout the body. In a crystal the atoms may be thought of as occurring in small groups, all groups being exactly alike, similarly oriented, and regularly aligned in all three dimensions. Each group can be regarded as bounded by a parallelepiped, and each parallelepiped regarded as one of the ultimate building blocks of the crystal. The crystal is formed by stacking together in all three dimensions replicas of the basic parallelepiped without any spaces between them. Such a building block is called a unit cell. Since the choice of a particular set of atoms to form a unit cell is arbitrary, it is evident that there is a wide range of choices in the shapes and dimensions of the unit cell. In practice, that unit cell is selected which is most simply related to the actual crystal faces and X-ray reflections, and which has the symmetry of the crystal itself. Except in a few special cases, the unit cell has the smallest possible size.

In crystallography the properties of a crystal are described in terms of the natural coordinate system provided by the crystal itself. The axes of this natural system, indicated by the letters a, b, and c, are the edges of the unit cell. In a cubic crystal, these axes are of equal length and are mutually perpendicular; in a tetragonal crystal they are of unequal lengths and no two are mutually perpendicular. The faces of any crystal are all parallel to planes whose intercepts on the a, b, c axes are small multiples of unit distances or else infinity, in order that their reciprocals, when multiplied by a small common factor, are all small integers or zero. These are the indices of the planes. In this nomenclature we have, for example, faces (100), (010), (001), also called the a, b, c faces, respectively. In the orthorhombic, tetragonal, and cubic
The next code calculates the hysteresis voltage $V_H$ as a function of the input charge $q$ in the Matlab/Simulink model. The code direction is inside the block: %Main model% / Hysteresis model / $V_H=GMS(q)$

```
function [Vh, q1, q2, q3, q4, q5, q6, q7, q8, q9, q10, q11] = fcn(q, qant, qb1, qb2, qb3, qb4, qb5, qb6, qb7, qb8, qb9, qb10, qb11)
% This function determines the instant position (or charge) of the n elements
% and the output hysteretic voltage
% Created by: Mario Quant, May 2009

qb = [qb1, qb2, qb3, qb4, qb5, qb6, qb7, qb8, qb9, qb10, qb11];
% Instant massless block positions for 11 elements

% Initial raising curve data taken from experimental characterization
% The Charge and Voltage values are processed to obtain the
% Brake voltage (v) and Capacitance value (C)
v = [9.9600E-02 1.0524E+00 3.9648E+00 7.6352E+00 9.1600E+00 1.1088E+01 1.2312E+01
     1.5552E+01 2.8800E+01 1.7856E+01 8.7360E+01];
C = [1.0442E-05 2.9647E-06 2.0985E-06 2.1794E-06 2.8384E-06 3.2828E-06 3.8012E-06
     3.6111E-06 2.1667E-06 3.6111E-06 7.7381E-07];

I = (q - qant);  % Determines the sign of the current
V = 0;           % Resets the output voltage

% This cycle determines each block position and the output voltage
% for each element
for j = 1:1:11
  if (abs((q - qb(j))/C(j))) < v(j)
    Vj = (q - qb(j))/C(j);
  else
    Vj = v(j)*sign(I);
    qb(j) = q - C(j)*v(j)*sign(I);
  end
  V = V + Vj;
end
Vh = V;          % Output values
q1 = qb(1);      
q2 = qb(2);      
q3 = qb(3);      
q4 = qb(4);      
q5 = qb(5);      
q6 = qb(6);      
q7 = qb(7);      
q8 = qb(8);      
q9 = qb(9);      
q10 = qb(10);    
q11 = qb(11);
```
Appendix C – Load Cell Characterization

The Load Cell used in the experiment was characterized using an Oscilloscope and a Force Dynamometer to obtain the characteristic curve for a 10V DC input.

<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>PA6110</td>
</tr>
<tr>
<td>Capacity</td>
<td>25 lb</td>
</tr>
<tr>
<td>Serial No.</td>
<td>552257</td>
</tr>
<tr>
<td>Output</td>
<td>3.0005 mV/V</td>
</tr>
<tr>
<td>Zero</td>
<td>&lt; 2% of Full Scale</td>
</tr>
<tr>
<td>Creep (20 minutes)</td>
<td>&lt; 0.03% of Full Scale</td>
</tr>
<tr>
<td>Non-Linearity</td>
<td>&lt; 0.03% of Full Scale</td>
</tr>
<tr>
<td>Hysteresis</td>
<td>&lt; 0.03% of Full Scale</td>
</tr>
<tr>
<td>Repeatability</td>
<td>&lt; 0.02% of Full Scale</td>
</tr>
<tr>
<td>Temp. Effect on Output</td>
<td>&lt; 15 PPM/°C Applied Load</td>
</tr>
<tr>
<td>Temp. Effect on Zero</td>
<td>&lt; 20 PPM/°C Applied Load</td>
</tr>
<tr>
<td>Operating Temp. Range</td>
<td>-40 °C to 80°C</td>
</tr>
<tr>
<td>Compensated Temp. Range</td>
<td>-10 °C to 40°C</td>
</tr>
<tr>
<td>Safe Overload</td>
<td>1.5 x Capacity</td>
</tr>
<tr>
<td>Input Impedance</td>
<td>385 ± 20 W</td>
</tr>
<tr>
<td>Output Impedance</td>
<td>350 ± 3 W</td>
</tr>
<tr>
<td>Insulation Resistance</td>
<td>&gt; 2,000 MW</td>
</tr>
<tr>
<td>Recommended Excitation</td>
<td>10V DC/AC</td>
</tr>
<tr>
<td>Maximum Excitation</td>
<td>20V DC/AC</td>
</tr>
</tbody>
</table>

Figure 62 - Load cell properties and characterization plot for 10V DC
Appendix D – Finite Element Modelling and Simulation in ANSYS

The FE model of a Cantilever Beam with a bonded piezoceramic patch actuator was modelled in ANSYS using batch programming. Details are first presented and the programmed code will afterward be shown.

**Element SHELL91:** Both the beam and actuator were modelled using shell elements. This type of element may be used for layered applications of a structure, allowing up to 100 layers.

**Static Analysis:** This analysis was programmed to analyze static reactions (tip displacement or strain in the piezoceramic) due to a force applied at the tip or a pair of moments generated by the PA following equation (45).

**Harmonic Analysis:** This analysis is used to verify the natural frequency of the modelled system. A frequency sweep sinusoidal force with defined amplitude acts on the beam tip. A frequency response plot can be obtained.

**Transient Analysis:** Dynamic behaviour can be observed with this analysis. Dynamic loads need first to be defined using functions, and different measures can be obtained.

Figure 63 presents the FE model with the boundary conditions. The beam is fixed at one end and pair of moments is applied at the border limits of the PA. The principal simulations are shown in Figure 53 and Figure 64, where piezoceramic strain and beam vertical displacement are measured due to application of forces with different amplitudes.

![Figure 63 - FEM elements and boundary conditions](image)
Batch code program:

!--------------------------------------------------------------------------------
!UNITS IN Newton, kilogram, meter, second
/COM, Mario Quant FEM Cantilever Beam with Piezo - May 2009
/PREP7
/TITLE, Cantilever Beam and Piezo
SMRT,OFF

ANTYPE,STATIC  !Three types of analysis available
!ANTYPE,TRANSIENT
!ANTYPE,HARMONIC
!============= VARIABLES =========================
tb=0.001   !beam thickness
lb=0.22   !beam length
wb=0.0381   !beam width
Eb=7.31E10   !beam Young Modulus
ta=0.000762   !actuator thickness
wa=0.0381   !actuator width
la=0.0508   !actuator length
E11=6.9E10   !actuator Young E11
d31=-220E-12  !PZT d31 coefficient
x1=0.01   !Distance of PZT positioning from base
tl=0.000254   !layer thickness (Of piezoceramic material)
w1=0.033274   !layer width
l1=0.045974   !layer length

!.... Static ....
V=-200  !Applied voltage
FL=(V*d31*w1*E11)/2 !Actuator force
M=(FL/2)*((ta+tb)/2) !Bending moment at two points using two actuators with 2 layers each
Fts=-2  !Force at the tip of beam

!.... Harmonic ....
Fth=1  !Applied tip force in Z
fmin=20  !Min. frequency in Hz for Harmonic analysis
fmax=40  !Max. frequency in Hz
ssteph=(fmax-fmin)*1 !Number of Steps

!.... Transient ....
freq=45 !Frequency for transient analysis
cycle=10 !Number of cycles to display
sstept=cycle*20 !Number of substeps
stime=1/freq*cycle !Time at end of substep

! ============= MODELLING ==============
!-------------------- Beam -------------------------
ET,1,SHELL91,16,,,0,4,1 !Element Type 1: Shell91
KEYOPT,1,8,1 !Store data for all layers
KEYOPT,1,9,0 !Do not use sandwich option
KEYOPT,1,11,0 !Nodes located at middle surface

K,1,0,0 !Keypoints
K,2,0,wb
K,3,x1,wb
K,4,x1,0
K,5,x1+a,0
K,6,x1+a,wb
K,7,lb,wb
K,8,lb,0

MP,EX,1,Eb !Material properties for Aluminum
MP,PRXY,1,0.33
MP,DENS,1,2768

MP,EX,2,E11 !PZT material properties
MP,PRXY,2,0.31
MP,DENS,2,7700
MP,GYZ,2,2.26E10

R,1,1 !Real Values for the first meshed area
RMORE
RMORE,1,0,tb !Layer 1 thickness
A,1,2,3,4
AATT,1,1,1,0
ESIZE,0.01
AMESH,1

R,2,2 !2 layers for the second meshed area
RMORE
RMORE,2,0,ta !Layer 1 thickness
RMORE,1,0,tb !Layer 2 thickness
A,3,4,5,6
AATT,2,1,0
ESIZE,0.01
AMESH,2

R,3,1 !Third meshed area
RMORE
RMORE,1,0,tb
A,5,6,7,8
AATT,3,1,1,0
ESIZE,0.01
AMESH,3

NSEL,S,LOC,X,0 !Select Fixed end
D,ALL,ALL !Fix one end of cantilever

NSEL,S,LOC,Y,0.5 !Select new subset of nodes
DSYM, SYMM, Y           !Symmetry plane down centre-line

!================================== ANALYSIS ===================================!
!------------------ Static ------------------
PSTRES, OFF         !Prestress for subsequent analysis

NSEL, S, LOC, X, x1  !Select load nodes and apply Moment
F, ALL, MY, -M/9     !9=Number of nodes in the y axis
NSEL, S, LOC, X, x1+a
F, ALL, MY, M/9      !Moment along end line of Piezo

NSEL, S, LOC, X, lb  !Applied Force at the Tip
NSEL, R, LOC, Y, (wb/2)
NSEL, R, LOC, Z, 0
F, ALL, FZ, Fts

NSEL, ALL          
FINISH

/SOLU               !Enters the solution processor.
OUTPR, BASIC, 1

SOLVE
SAVE
FINISH             !Exits normally from a processor.

/POST1
ETABLE, DisplZ, U, Z  !Component Z structural displacement
ETABLE, StrainX, EPEL, X !Component elastic strain in X

/OUTPUT, T1Bresults, dat !Redirects text output to a file or to the screen
PRNSOL, U            !Prints the nodal solution results
PLNSOL, U, Z, 2      !Displays results as continuous contours
PRESOL, SMISC, 1     !Prints the solution results for elements

/OUTPUT
FINISH

!------------------ Harmonic ------------------
!PSTRES, OFF         !Prestress for subsequent analysis

!NSEL, ALL           !Deletes previous forces
!FDELETE, ALL, ALL

!NSEL, S, LOC, X, lb  !Applied Force at the Tip
!NSEL, R, LOC, Y, (wb/2)
!NSEL, R, LOC, Z, 0
!F, ALL, FZ, Fth
!NSEL, ALL

!HARFRQ, fmin, fmax  !Defines the frequency range (Hz)
!NSUBST, ssteph      !Specifies the number of substeps
!KBC, 1              !Specifies stepped or ramped loading

!FINISH

!/SOLU
!SOLVE
!SAVE
!FINISH

!/POST26
!NSOL, 2, 152, U, Z, UZ_2 ! Create variable #2 for tip displacement
!/OUT
!/TITLE, HARMONIC RESPONSE OF BEAM UNDER LOAD AT TIP
!/AXLAB, Y, TIP TRANSVERSE DISPLACEMENT AMPLITUDE
!PLVAR, 2 ! Graph variable #2
!FINISH

!---------------- Transient (Dynamic) ----------------
!NSEL, S, LOC, X, lb ! Applied force at the Tip
!NSEL, R, LOC, Y, (wb/2)
!NSEL, R, LOC, Z, 0
!F, ALL, FZ, %SHAKER%

!NSEL, S, LOC, X, x1 ! Select load nodes and apply Moment (Maximum Moment)
!F, ALL, MY, %PZTA%
!NSEL, S, LOC, X, x1+la
!F, ALL, MY, %PZTB%

!NSEL, ALL
!SAVE
!FINISH

!/SOLU
!TRNOPT, FULL,,, HHT
!SOLCONTROL, ON ! Specifies whether to use optimized nonlinear solution
!TIMINT, ON, ALL ! Include transient (mass or inertial) effects
!OUTRES, ALL, ALL ! Controls the solution data written to the database
!KBC, 1 ! Specifies stepped or ramped loading
!NSUBST, sstep ! Specifies the number of substeps
!OUTPR, ALL, ALL ! Print solution for this item for every substep
!TIME, stime ! Sets the time for a load step
!SOLVE
!FINISH

!/POST26
!NSOL, 2, 152, U, Z, UZ_2
!/OUT
!/TITLE, TRANSIENT RESPONSE OF BEAM UNDER LOAD
!/AXLAB, Y, TIP TRANSVERSE DISPLACEMENT AMPLITUDE
!PLVAR, 2
!FINISH

!--------------------------------------------------------------------------------
Appendix E – Test Bench Design

A test bench was designed in CAD and built to measure the performance curves of the piezoceramic.

Figure 65 - CAD design for test bench

Figure 66 - CAD design for base 1
Figure 67 - CAD design for base 2

Figure 68 - CAD design for base 3